Intergenerational Contracts and Female Labor Supply*

Yang Yue †
Arizona State University

December 2, 2016

Abstract

This article examines how intergenerational relationships between parents and grandparents affect females' labor supply. I develop a non-altruistic dynamic contract model using economic benefits such as a bequest, coinsurance, and cheaper care service to sustain such relationships in the face of long-term incentive problems. I then estimate the parameters of the model using Chinese household surveys. I evaluate the labor and income reallocation effect throughout the relationships. I find that intergenerational relationships increase the labor supply of younger females by 32% but reduce the labor supply of older females by 21%, while increasing older females' household savings by 13%. My policy experiments produce the following predictions: delaying retirement age reduces the labor supply of young females; raising inheritance taxes increases the labor supply of young females and savings of both parents and grandparents. Therefore, I find that a public policy affects the households attached to the target group through intergenerational relationships.

Key Words: dynamic contract, intergenerational relationship, female labor supply, income transfer, elder support, elder care, child care, bequest, retirement, bequest

1 Introduction

This article examines the influence of intergenerational relationships on females' labor supply decisions and households' reactions to a number of policies. Intergenerational relationships build economic connections between households of various generations. In the relationships, households of different generations

*I am grateful to my advisors Daniel Silverman, Natalia Kovrijnykh, Gregory Veramendi, and Matthew Wiswall, for their guidance and support at all stages of this research project. Any errors are my own.

†Department of Economics, WP Carey School of Business, Arizona State University. *Tel.:* 1-480 374 0917 *E-mail address:* yyue7@asu.edu

provide income transfer, child care and elder care to each other. These activities affect households' labor supply and saving decisions by redistributing labor and income across generations. Furthermore, economic ties matter in evaluating the effects of government policies. Through intergenerational relationships, public policies are not only affecting the target groups, but also affecting the households attached to the target groups. In countries without strong social welfare programs[1], intergenerational relationships are the main way to provide elder support, child care and family insurance. For example, 66% of the Chinese elderly (aged 65 and over) provides child care to their grandchildren (Wu et al., 2014), 45% of the Chinese elderly live with their children and 22% of their income come from their children (National Survey Research Center, 2014). In these countries, intergenerational relationships substantially reshape households' decisions on savings and labor supply, as well as policy implications of various public policies.

I construct a theoretical framework to analyze the incentive problems in intergenerational relationships. In an intergenerational relationship, households exchange income and labor service in different periods. The exchange is not balanced within each period. Without incentives to keep households committed to the relationship at each stage, the sequential exchange will not happen from the beginning. For example, grandparents take care of their grandchildren to exchange parents' elder support in the future. However, when grandparents are old, parents refuse to support grandparents, if they cannot get benefits from it. Without the confirmation of payback, grandparents will not help parents from the beginning. Considering intergenerational relationships provide incentives for the sequential exchange is a prerequisite for evaluating the impact of intergenerational relationships on household behavior. I build a dynamic contract model using economic benefits to keep the households in the relationships.

I quantify the effect of intergenerational relationships on females' labor supply over the life cycle. The relationships reallocate labor across generations by affecting households' child and elder care decisions. Furthermore, grandparents' help free young parents from child care and enable them to stay in their jobs. However, to assist parents, grandparents may work less and leave their jobs early. Moreover, as grandparents grow old, parents provide elder care service to grandparents and work less. An intergenerational relationship has various effects on females' labor supply at various stages. I use a structural model to estimate the labor reallocation effects of each stage of the relationship.

I measure the spillover effects of public policies through intergenerational relationships. The spillover effects can change the policy implications of public policies. For example, a government delays the mandatory retirement age to increase seniors' labor supply. Then, grandparents, who used to take care of their grandchildren, need to get back to work. As the result, parents reduce their working time to take care of their children. Thus, delaying mandatory retirement age reduces the labor supply of young females through intergenerational relationships. Ignoring the economic connections between generations

[1] Table 5 in the Appendix compares intergenerational relationships across countries.

could lead to incomplete forecasts of the impact of some policy change. I conduct policy experiments to evaluate various public policies with the existence of intergenerational relationships.

The first contribution of the article is to develop a non-altruistic dynamic contract framework that uses economic benefits to sustain long term intergenerational relationships. The contract model is built by adding labor supply and preference for child and elder care, as well as bequest to the Kocherlakota (1996) environment[2]. The dynamic contract framework solves the incentive problems of the relationships and creates Pareto improvements for the two households by exchanging income and labor service. The economic benefits from labor allocation, risk sharing, and bequests give households utility values higher than those of outside options at any stage. Furthermore, the Pareto gains provide incentives for the households to remain in the relationships. The model also reveals the effects of incentive problems on household behaviors. Without commitment problem, the two households with full-commitment behave like a single household, in which two households have the same preference for the parents' utility relative to the grandparents' utility. So, I interpret the full-commitment contract model as a perfect altruistic model. I can use the dynamic contract framework to derive of the rules of income and labor service reallocation in intergenerational relationships. On the basis of the rule, I identify the effect of intergenerational relationships on savings and labor supply. These expressions enable the formal identification proof and promote the estimation of the structural parameters.

The second contribution of the article is to quantify the effect of intergenerational relationships on the female labor supply in China. I implement the identification strategy and estimate the model using data from Chinese household surveys. I choose these surveys for three major reasons. First, China has strong intergenerational relationships. Observing and estimating the influence of relationships on households' decisions is easy. Second, several major social policy changes have occurred in recent decades. These changes provide enough variations to identify the effects of economic condition changes on intergenerational relationships is easy. Finally, these surveys have detailed information about income transfer, child care and elder care. The information enables us to identify the economic connections between households. I can use this information to measure the extent to how households conduct child and elder care, as well as transfer decisions. The results show that grandparents taking care of their grandchildren increases the labor supply of parents by 32%, but decreases the labor supply of grandparents by 21% in the earlier stage of the relationships. In providing elder care to grandparents, parents reduce their labor supply by 13% at the later stage of the relationships. The wage structure in China contributes to the strong effects of intergenerational relationships on labor supply. Given that younger generations have a higher wage rate, the total income of two households increases when grandparents take care of their grandchildren

[2] Kocherlakota (1996) built a dynamic risk sharing problem between two risk averse agents living in infinite horizon and facing idiosyncratic income shocks. In the paper, risk sharing is limited by two-sided lack of commitment to the insurance contract.

and free parents to work. I also compare the differences of households' behavior between the contracts with and without incentives problem. I found a bad income shock for grandparents, for example, can increase the transfer and support from parents to grandparents in coming periods in the contract without incentive problems, but decreases that in the contract with incentive problems.

The third contribution of the article is to evaluate the spillover effects of various policies through intergenerational relationships. My policy experiments reveal the effect of social welfare programs on female labor supply and savings as well as on households' transfer, elder and child care decisions. I impose a 20% subsidy on the child care and elder care services from market. I find that 20% of child care subsidy increases grandparents' labor by 41% and 20% elder care subsidy increases parents' labor supply by 13%. I then increase the inheritance tax from 0% in benchmark to 30%. The direct effects of inheritance tax reduce grandparents' bequests in exchange for parents' labor service and income transfer. Parents reduce the income transfer and labor support to grandparents. Grandparents increase their savings to maintain the bequest incentives. The change increases grandparents' savings by 14% and increases parents' labor supply by 9%. Finally, I delay the mandatory retirement age from 60 in benchmark to 65. I find that pushing back the mandatory retirement age reduces the parents' labor by only 8% at the delaying period when grandparents have a lower wage rate than parents. The wage structure determines that grandparents are always the child care providers before and after the policy change. Then I change the wage structure by reducing the wage gap between parents and grandparents. I find that delaying mandatory retirement age decreases the labor supply of parents by 27% at the delaying period. With the new wage structure, grandparents no longer provide child care. Therefore, the spillover effects through intergenerational relationships change the implications of these public policies.

2 Literature

2.1 Motives of Intergenerational Relationships

Since Barro (1974) and Becker (1974), researchers mainly use altruism[3] to address the incentive problem in intergenerational relationships. Altruism sustains long term intergenerational relationships, but cannot explain several phenomena. For example, parents account for the relative economic positions of their children and transfer wealth to or share their inheritance with their children unequally (Schanzenbach & Sitkoff 2008). Although altruistic parents are expected to give more to their less well-off children, bequests tend to divide equally among siblings (McGarry 2001). As altruism is morally charged, it should be independent of institutions and economic factors. Many studies have found that family ties in countries with weak social welfare programs tend to be stronger

[3] Altruism indicates that a parent (child) can derive utility from the consumption of his child (parent).

than those countries with strong programs (Bonsang 2007; Hank & Buber 2008). Altruistic models fail to explain the relationship of intergenerational ties with these economic factors. I use a non-altruistic model to emphasize the functions of observable economic factors on intergenerational relationships.

Some studies use non-altruistic forces to address the incentive problem in intergenerational relationships. Researchers have argued that households enforce the intergenerational ties through a self-enforcing constitution[4] (Cigno 2006), demonstration effect (Jellal & Wolff 2005) or nurtured altruism (Stark & Zhang 2002). However, these studies ignore the direct economic benefits from intergenerational relationships. Direct economic benefits can sustain intergenerational relationships. Intergenerational relationships improve labor allocation efficiency by allowing those who are more productive to work (Geurts, van Tilburg, Poortman & Dykstra 2015). Furthermore, assets after unexpected death are an important source of bequest (Lockwood 2014). Accidental bequest has no direct effect on parents' utility, but is used in exchange for children's transfer and support. The Pareto improvement through risk sharing, cheap care service and bequest creates economic benefits for intergenerational relationships. My model combines these direct economic benefits to sustain intergenerational relationships.

2.2 Female Labor Supply Affected by Intergenerational Relationships

The provision of child and elder care reduces females' labor supply. Mothers face the problem of reconciling work and child care responsibilities. Grandparents may substitute for mothers in doing child care work and thus mothers become free to work. Grandparents' assistance strongly increases younger parents' labor supply (Posadas & Vidal-Fernandez 2012; Compton & Pollak 2014, Battistin, Nadai &Padula, 2015). Within intergenerational relationships, the question of who provides child and elder care is primarily determined by individuals' health and income conditions. Working grandparents use more money to subsidize their grandchildren rather than provide care directly (Luo, LaPierre Hughes & Waite, 2014). Grandparents with newborn grandchildren are more likely to provide care for their grandchildren, and married grandparents are also more likely to work and to provide financial help (Ho 2015). In the later periods of intergenerational relationships, children provide income support and physical care to their parents. In most developing countries, most of the elderly population receives financial support from their adult children (Hamaaki, Hori & Murata 2014). Parents who provide care for grandparents cause the great reduction of female labor supply at midlife (Johnson & Sasso 2006). The literature focuses mostly on the effects of intergenerational relationships in a single period. Conversely, this article determines the influence of intergenerational relationships on females'

[4] Cigno (2006) relies on social norms explain long-term relationships. Families can be viewed as communities governed by self-enforcing constitutions. In the OLG framework, if people do not support their parents, their children will be not willing to support them. In this environment, the dominant strategy of individuals is to provide transfer and help to other people.

labor supply throughout the life cycle.

Formal care service and social welfare programs can substitute for intergenerational relationships by providing the same service to households. The expansion of public child care provokes a large positive effect on maternal employment (Bauernschuster & Schlotter 2015). Married women's labor supply decreased with the ascending cost of formal child care (Blau & Kahn 2005). Elder care giving decreases female work intensity and the impact decreases after the launch of the market oriented elder care insurance (Sugawara & Nakamura 2014). The Temporary Assistance for Needy Families program undermines intergenerational support (McDonald & Armstrong 2001). The implementation of National Health Insurance in Taiwan in 1995 decreased the likelihood of intergenerational coresidency (Hsieh, Chou, Liu & Lien 2015). In this article, formal care service and social welfare programs affect the outside options of intergenerational relationships.

3 Background

China has a high female labor force participation rate. The high female labor force participation rate is a legacy of the Communist Party's rule that women are equal to men in all spheres of life (Yu & Liu, 2010). Yet, since the 1980s, the transition to market economy has widened the gender income gap[5]. Between 1990 and 2014, Chinese females' labor force participation rates declined from 77% to 64%. But, the rate is still higher than the world average of 50% (World Bank 2016).

Strong intergenerational connections increase the labor supply of young females (Chen & Liu 2009). Grandparents who care their grandchildren are common in most families in China (Chen, Liu & Mair 2011). Grandparents provided child care to grandchildren in 35% of family setups in rural China (Silverstein, Cong, & Li 2006). In 2010, about 66% of people older than 60 years of ages have provided care for their grandchildren (Melenberg & Zheng 2012). The provision of child care by grandparents affects both parents' and grandparents' labor supply decisions. For example, the participation of daughters' in the labor force is one major reason why grandmothers provide child care (Chen, Liu, & Mair 2011). Traditionally, children in China bear the ultimate responsibility for taking care of their aging parents (Chen & Liu 2009). An adult child faces criminal charges for refusing to support an aged parent[6]. In China, elders in most areas do not have a formal safety net. The majority depends exclusively on their children for support (Cong & Silverstein 2012). About 45% of people older than 60 years live with their children and 22% of their income comes from their children (National Survey Research Center, 2014).

[5] According to National Bureau of Statistics of China, in 1990, the average female's wage rate was 78% in rural area and 79% in urban area of that of the average male. In 2013, the ratio declined to 67% in urban areas and 57% in rural areas. .

[6] The Chinese constitution of 1982 proclaims the obligation of adult children to support their elderly parents.

The low fertility rate caused by the one child policy[7] contributes to the high female labor participation rate. In a unique "four-two-one" family[8], the only child receives child care from four grandparents in childhood, and also bears the responsibility of supporting two parents and, sometimes, four grandparents in their old age. Since the enforcement of the one child policy in 1980s, the fertility rate in China decreased from 3 in 1980 to 1.6 in 2015, which is lower than the world average of 2.6 (World Bank 2015). The low fertility rate reduces females' burden of child care and causes a high female labor market participation rate.

Weak institutionalized care and social welfare programs also contribute to the high female labor participation rate. After China's economic transition in the 1980s, publicly funded care[9] and elder care[10] is largely eliminated, and market care service is either too expensive for most households to afford or suffer from low quality (Zhang & Maclean 2012). In 2003, accordance with the National Research Center on Aging, only 2% of the population aged 65 and over use institutionalized care. In addition, only 46% of urban employees were covered by a pension plan in 2004 (Trinh 2006), and only 12% of the rural labor force participated in the old age social insurance programs scheme in 2006 (Wang 2006). Without strong institutionalized care and social welfare programs, households can only rely on intergenerational relationships to provide child care and elder support.

4 Theoretical Model

In this section, I present a dynamic contract model of inter-household decision making. Households can live independently, or join a contract through the mutual provision of a series of state contingent income transfers, elder care, and child care to each other. I define the case without interactions as the autarky case, and that with interactions as contracts. Households remain in the contract, purely because of economic benefits they can gain. In contrast to the existing literature, the model does not take altruism into consideration. Ignoring altruism does not mean denying the importance of altruism, but doing so highlights the functions of observable economic factors in intergenerational relationships.

The model has two agents: parent and grandparent. The parent's household has one child, who makes no decisions and needs child care service. To simplify, I only look at the female supply decisions, and take the male's labor supply and

[7] The one-child policy, introduced in 1979, only allowed families to have one child each. Since 1984, a rural family can have a second child if the firstborn is female (Chen, Jin & Yue 2010). Since 2014, all couples can have second children. See Figure 6 in the Appendix for the details of the fertility rate change.

[8] In a "Four-two-one" family, the child is the only child for two parents and the only grandchild for four grandparents.

[9] The number of publicly funded kindergartens dropped from about 150,000 (an 83% market shares) in 1998 to about 43,000 (a 24% market share) by in 2012 (National Bureau of Statistics 2016).

[10] China has just about 2% of people ages sixty-five and older living in residential care facilities. (Feng, Liu, Guan & Mor 2012).

income as exogenous. For notational convenience, I denote the age of the parent and the calendar year by t. I assume the grandparent is 6 periods older than the parent. At period t, the parent's age is t and the grandparent's age is t+6. An agent lives for 20 periods at most. At age t, an agent's death rate is ϱ_t. At time t, the parent's death rate is ϱ_t and the grandparent's death rate is ϱ_{t+6}. In simplifying the model, the parent's death rate is 0 before period 14. The parent will not die before the grandparent. The agent needs child care from age 1 to 4. From age 11 to 20, the agent needs elder care. The agent retires after period 9. After age 10, the agent no longer provides elder and child care as well as work[11].

4.1 Preferences

Household i has preference on consumption c_t^i, leisure l_t^i, child care hours K_t^i, and elder care hours N_t^i. Child care service can come from the parent k_t^p, the grandparent k_t^g, or an outside service k_t^m, with $K_t^i = k_t^p + k_t^g + k_t^m$. Elder care service can come from parent n_t^p, and outside service n_t^o, with $N_t^i = n_t^p + n_t^m$. η represents the preference over leisure. α_t represents the preference over child care. γ_t represents the preference over elder care. α_t and γ_t, the tastes over child care and elder care service, change over time. α_t equals to 0 after age 4. γ_t equals to 0 when the household is younger than age 10. At time t, the parent's utility parameters are $\theta_t, \eta_t, \alpha_t$ and γ_t; the grandparent's utility parameters are, θ_{t+6}, η_{t+6}, α_{t+6}, and γ_{t+6}. Agent i's current-period utility function at age t is:

$$U_{it}(c_t^i, l_t^i, K_t^i, N_t^i) = \ln c_t^i + \eta \ln l_t^i + \alpha_t \ln K_t^i + \gamma_t \ln N_t^i \tag{1}$$

4.2 Budget Sets

Households take the price of outside child care and elder care service p_t^k and p_t^n, wage rate w_t^i, and the interest rate R_t, as given. The budget constraint $\left(BC_t^i\right)$ is:

$$c_t^i + s_{t+1}^i + p_t^k k_t^m + p_t^n n_t^m + T_t^i \leq R_t s_t^i + w_t^i h_t^i + \epsilon_{kt}^i, \forall i \in \{p, g\} \qquad (BC_t^i)$$

The money endowment is from asset $R_t s_t^i$, wage income $w_t^i h_t^i$, which is determined by wage rate w_t^i and working hours h_t^i, and a random income shock[12] ϵ_{lt}^i. ϵ_{jt}^i with probability π_{tj}^p, $\sum_{j=1}^{J} \pi_{jt}^p = 1$, and $\epsilon_{jt}^p \in \{\epsilon_{1t}^p, ..., \epsilon_{Jt}^p\}$; ϵ_z^g with probability π_{zt}^g, $\sum_{z=1}^{Z} \pi_{zt}^g = 1$, and $\epsilon_{zt}^g \in \{\epsilon_{1t}^g, ..., \epsilon_{Zt}^g\}$. I assume, $w_t^p + \epsilon_{1t}^p > 0$,and $w_t^g + \epsilon_{1t}^g > 0$. The assumption avoids corner solutions. Household i spends the

[11] Retired agent can still provice child care at age 10.

[12] In this paper, income shocks are from the uncertainty of not working income and health spending. For example, a man has a certain probability of getting a disease every year. If he gets sick, he needs to pay for medical treatment, which is a bad income shock for him. If he is healthy, he doesn't need to spend money on medical treatment, which is a good income shock for him.

money on consumption c_t^i, saving for the next period s_{t+1}^i, outside child care k_t^o and elder care n_t^o, and net transfer to the other household T_t^i.

4.3 Time Allocation

Each household's overall time is 1. Household i can spend time on work h_t^i, leisure l_t^i, elder care n_t^i and child care k_t^i. The time constraint $\left(TC_t^i\right)$ is:

$$1 = h_t^i + l_t^i + n_t^i + k_t^i, \forall i \in \{p, g\}. \qquad (TC_t^i)$$

4.4 Autarky Case

I define the en-ante value function in autarky case and call this autarky value $V_{it}^{aut}(s_t^i)$. After the income shock is realized, each household makes an optimal allocation of current-period consumption, individual savings s_{t+1}^i, carried on to the next period, labor-force participation decision h_t^i, child care decisions k_t^i and k_t^o, and elder care decisions n_t^i and n_t^o, which solves the following optimally constrained problem. Define the set of decisions made as $\Omega_{ti} = \left\{s_{t+1}^i, h_t^i, n_t^i, k_t^i, n_t^o, k_t^o\right\}$. The Bellman equation of the autarky case is[13]:

$$v_t(s_t^i, \epsilon_{kt}^i) = \max_{\Omega_{ti}} U_{it}(c_t^i, l_t^i, K_t^i, N_t^i) + \beta\left(1 - \varrho_{t+1}^i\right) V_{t+1}^{aut}(s_{t+1}^i), \forall i \in \{p, g\} \qquad (2)$$

subject to budget constraint BC_t^i and time constraint TC_t^i. In this case, the solution to the problem above yields the following expected value function for the household at the beginning of period t. The autarky value is:

$$V_{it}^{aut}(s_t^i) = \sum_{j=1}^{J} \pi_{jt}^i v_t(s_t^i, \epsilon_{jt}^i) \qquad (3)$$

4.5 The Optimal Contract

A contract determines on bequest, punishment, income transfer, child and elder care, savings and consumption. A contract can end in two cases. If one side breaks the rule of the contract, the contract will end as a punishment. If the grandparent dies in the contract, the contract will end. At the same time, the parent receives the grandparent's savings as a bequest and lives in the autarky case. In the contract, each household leaves the relationship at any time. I call this the no-commitment case. I also define a special case that neither household can leave after both households join in the contract. I call this the full-commitment case. No incentive problems exist in this case. I use full-commitment contract as a benchmark to show the first-order importance of the incentive problems.

In the initial period t=0 of a contract, both households choose to either join in the contract or live alone. If households cannot reach an agreement, each

[13] ϱ_{t+1}^i is the death rate of agent i, with $\varrho_{t+1}^p = \varrho_{t+1}$ and $\varrho_{t+1}^i = \varrho_{t+7}$.

household will live in the autarky case forever after period 0. If they reach an agreement, they go to the next period. Before period t=1 and onwards, the death shock happens. If the grandparent dies, the parent receives the all the savings of the grandparent and lives in the autarky case. If neither household dies, both households enter the contract of the period. Taking the income as given, the contract gives the promised support and transfer to each other, as well as makes the consumption, time allocation and savings decisions. (see Figure 1 in the Appendix for the timing of the contract)

The rule of income and labor service allocation is adjusted to an allocation that lies on the Pareto frontier. The state space is comprised of each household's asset s_t^i and the grandparent's promised value G_t. I define the parent's value function $P_t(s_t^p, s_t^g, G_t)$. In solving for the optimal contract, one maximizes $P_t(s_t^p, s_t^g, G_t)$ subject to delivering at least G_t the grandparent. Following Spear and Srivastava (1987), one can rewrite the sequence problem corresponding to the optimal contract in recursive form, with the promised value as a state variable, and continuation values to the grandparent as control variables. Essentially, the promised value summarizes the previous history of the play.

4.5.1 No-commitment Case

A contract determines the net transfer T_{tjz}^i, with $T_{tjz}^p = -T_{tjz}^g$, child care support k_{tjz}^g, and elder care support n_{tjz}^p in the state with income shocks ϵ_j^p and ϵ_z^g. I define the set of decisions made on support and transfer as $\Gamma_{tjz} = \left\{n_{tjz}^p, k_{tjz}^g, T_{tjz}^p\right\}$. According to the contract, contracts makes an optimal allocation of current-period consumption c_{tjz}^p and c_{tjz}^g, as well as the time allocations to each household such as labor-force participation decision h_{tjz}^i, child care decisions k_{tjz}^i and k_{tjz}^m, elder care decisions n_{tjz}^i and n_{tjz}^m. I define the set of decisions made on care giving as $\digamma_{tjz} = \left\{h_{tjz}^g, n_{tjz}^m, h_{tjz}^p, k_{tjz}^p, k_{tjz}^m\right\}$. In addition, the contract determines the individual savings s_{t+1jz}^i, and the promised value G_{jz}^{t+1}. I define the set of decisions made on state variables as $\Omega_{tjz} = \left\{s_{t+1jz}^g, s_{t+1jz}^p, G_{jz}^{t+1}\right\}$. The Bellman equation is:

$$P^t(s_t^p, s_t^g, G^t) = \max_{\{\digamma_{tjz}, \Omega_{tjz}, \Gamma_{tjz}\}} \sum_{j=1}^{J}\sum_{z=1}^{Z} \pi_{jt}^p \pi_{zt}^g [U_{pt}(c_{tjz}^p, l_{tjz}^p, K_{tjz}^p, N_{tjz}^p)$$
$$+\beta\left(1-\varrho_{t+7}\right) P^{t+1}(s_{t+1jz}^p, s_{t+1jz}^g, G_{jz}^{t+1}) + \beta\varrho_{t+7} V_{pt+1}^{aut}(s_{t+1jz}^p + s_{t+1jz}^g)]$$

subject to budget constraints BC_t^p and BC_t^g, time constraints TC_t^p and TC_t^g, the promise keeping constraint at period t:

$$\sum_{j=1}^{J}\sum_{z=1}^{Z} \pi_{jt}^p \pi_{zt}^g \left[U_{gt}(c_{tjz}^g, l_{tjz}^g, K_{tjz}^g, N_{tjz}^g) + \beta\left(1-\varrho_{t+7}\right) G_{jz}^{t+1}\right] \geq G^t \qquad (PK_t^g)$$

the incentive constraints[14] given any income shocks at period t:

$$U_{gt}(c^g_{tjz}, l^g_{tjz}, K^g_{tjz}, N^g_{tjz}) + \beta\left(1-\varrho_{t+7}\right) G^{t+1}_{jz} \geq v^{aut}_{gt}(s^g_t, \epsilon^g_{zt}), for \forall j, z \quad (IC^g_t)$$

and

$$U_{pt}(c^p_{tjz}, l^p_{tjz}, K^p_{tjz}, N^p_{tjz}) + \beta\left(1-\varrho_{t+7}\right) P^{t+1}(s^p_{t+1jz}, s^g_{t+1jz}, G^{t+1}_{jz}) \\ +\beta\varrho_{t+7} V^{aut}_{pt+1}(s^p_{t+1jz} + s^g_{t+1jz}) \geq v^{aut}_{pt}(s^p_t, \epsilon^p_{jt}),\ for\ \forall j, z \quad (IC^p_t)$$

the participation constraints at period t+1:

$$G^{t+1}_{jz} \geq V^{aut}_{gt+1}(s^g_{t+1jz}), for \forall j, z \quad (PC^g_{t+1})$$

and

$$P^{t+1}(s^p_{t+1jz}, s^g_{t+1jz}, G^{t+1}_{jz}) \geq V^{aut}_{pt+1}(s^p_{t+1jz}),\ for\ \forall j, z \quad (PC^p_{t+1})$$

The promise-keeping constraint (PK^g_t) ensures that the contract delivers the promised level of discounted utility to the grandparent. It plays the role of a law of motion for the state variables. The incentive constraint for household i (IC^i_t) is the incentive compatibility constraint ensuring that the household i gets a higher ex-post utility value from the contract than it could from the autarky case. The participation constraint for household i (PC^i_{t+1}) is the incentive compatibility constraint ensuring that household i gets a higher ex-ante utility value from the contract, than it could from the autarky case in the next period.

4.5.2 Full-commitment Case

In a full-commitment case, no incentive problems exist. The Bellman equations of no-commitment contract doesn't have participation constraints, and incentive constraint.

4.6 Characterization of the Optimal Contract

4.6.1 Full-commitment Case

This section characterizes the optimal contract. λ_t is the Lagrangian multiplier associated with promise keeping constraint of the grandparent household. Using the envelope theorem, I get:

$$G^t : \frac{\partial P^{ft}(s^p_t, s^g_t, G^t)}{\partial G^t} = -\lambda_t \quad (ET1)$$

The first order conditions for the optimal contract problem are:

$$T^p_t \quad : \quad \frac{c^g_{tjz}}{c^p_{tjz}} = \lambda_t \quad (FOC2)$$

$$G^{t+1}_{jz} \quad : \quad \frac{\partial P^{f,t+1}(s^p_{t+1jz}, s^g_{t+1jz}, G^{t+1}_{jz})}{\partial G^{t+1}_{jz}} = -\lambda_t \quad (FOC3)$$

[14] The incentive constraints are indeed the ex-post participation constraints. To simplify the notation, I call these constraints as incentive constraints.

Proposition 1 *In the full-commitment case, both households fully share the income risk. The consumption ratio is a constant through the all periods.* $\frac{c^g_{tjz}}{c^p_{tjz}} = \lambda_t$, *with* λ_t *is a constant, for* $\forall t \in [1, T]$, $\forall \epsilon^p_{jt}$ *and* ϵ^g_{zt}*.(see Appendix for proof.)*

Two households fully share risks in full-commitment case. Equation ET1 defines the ex-ante income and labor service allocation rule. Equation FOC2 defines the ex-post income and labor service allocation in the state with income shocks ϵ^g_{zt} and ϵ^p_{jt}. Equation FOC3 defines the ex-ante income and labor service allocation rule of at time t+1. The three equations show that the full-commitment case always gives constant utility weights $\frac{\lambda_t}{\lambda_t+1}$ to the grandparent's utility, and $\frac{1}{\lambda_t+1}$ to the parent's utility at any state of any period. In addition, $\lambda_t = \lambda_1$ for $\forall\ t$. The utility weight on each term is a constant over time. To give an intuition of the result, consider two households maximizing their ex-ante utilities. The contract allows the household in a relatively good state to transfer its income and labor service to the household that is in a relatively bad state. Two households smooth their consumption by allocating a constant portion of the income and labor service to each other. Fully risk sharing increases both households' ex-ante utility values.

An optimal contract with full-commitment is equivalent to a single household problem with special altruistic preferences. In the single household problem, the parent and the grandparent have the same preference for the parent's utility relative to the grandparent's utility. The preference for each household becomes:

$$\nu_t = U_{pt}(c^p_{tjz}, l^p_{tjz}, K^p_{tjz}, N^p_{tjz}) + \lambda_1 U_{gt}(c^g_{tjz}, l^g_{tjz}, K^g_{tjz}, N^g_{tjz}) \tag{4}$$

The incentive problems in intergenerational relationships cause the differences of households' behaviors between the intergenerational contract model and altruistic models. Without the incentive problems, households in an optimal contract behave like altruistic households. The only difference between the contracts with full-commitment and altruistic problems is on the initial utility weight λ_1. λ_1 Is endogenous chosen in contract, but exogenous given in the problem with altruistic households. In this paper, I use the optimal contract with full-commitment to demonstrate households' behavior with altruistic preference.

4.6.2 No-commitment Case

I characterize the optimal contract using the first order conditions and envelope theorem. λ_t is the Lagrangian multiplier associated with the grandparent household's promise-keeping constraint. $\pi^p_{jt}\pi^g_{zt}\theta_{jz}\left(1-\varrho_{t+7}\right)$ are the Lagrangian multipliers associated with grandparent's participation constraint. $\pi^p_{jt}\pi^g_{zt}\mu_{jz}\left(1-\varrho_{t+7}\right)$ are the Lagrangian multipliers associated with the grandparent household's participation constraint. $\pi^p_{jt}\pi^g_{zt}\varkappa_{jz}$ are the Lagrangian multipliers associated with the parent household's participation constraint. $\pi^p_{jt}\pi^g_{zt}\omega_{jz}$ are the Lagrangian multipliers associated with the parent household's incentive constraints. Using the envelope theorem, I get:

$$G^t : \frac{\partial P^t(s^p_t, s^g_t, G^t)}{\partial G^t} = -\lambda_t \tag{ET2}$$

From the first order conditions (see Appendix for details) for the optimal contract problem, I get:

$$T_t^p \quad : \quad \frac{c_{tjz}^g}{c_{tjz}^p} = \frac{\lambda_t + \varkappa_{jz}}{1 + \omega_{jz}} \qquad (FOC4)$$

$$G_{jz}^{t+1} \quad : \quad \frac{\partial P^{t+1}(s_{t+1jz}^p, s_{t+1jz}^g, G_{jz}^{t+1})}{\partial G_{jz}^{t+1}} = -\frac{\lambda_t + \varkappa_{jz} + \theta_{jz}}{1 + \omega_{jz} + \mu_{jz}} \qquad (FOC5)$$

Equation ET2 defines the ex-ante rule of income and labor allocation. Equation FOC3 gives the ex-post rule of income and labor allocation, when the income shocks are ϵ_{zt}^g and ϵ_{jt}^p. Equation FOC5 defines the law of motion of the rule of income and labor allocation at time t+1. Different from the full-commitment case, the utility weights are no longer fixed. Income shocks change the ex-post utility weights of the current period and the ex-ante utility weights of the future. I obtain the optimal choices vectors $\{\digamma_{tjz}, \Omega_{tjz}, \Gamma_{tjz}\}$ from the optimal contract. I define the ex-post utility values of the optimal contract, given the income shock ϵ_{jt}^p and ϵ_{zt}^g, as follows:

$$\begin{aligned} p_t\left(s_t^p, s_t^g, \epsilon_j^p, \epsilon_z^g, G^t\right) &= U_{pt}(c_{tjz}^p, l_{tjz}^p, K_{tjz}^p, N_{tjz}^p) + \beta\varrho_{t+7}V_{pt+1}^{aut}(s_{t+1jz}^p + s_{t+1jz}^g) \\ &\quad + \beta\left(1 - \varrho_{t+7}\right)P^{t+1}(s_{t+1jz}^p, s_{t+1jz}^g, G_{jz}^{t+1}) \qquad (5) \end{aligned}$$

and

$$g_t\left(s_t^g, s_t^p, \epsilon_z^g, \epsilon_j^p, G^t\right) = U_{gt}(c_{tjz}^g, l_{tjz}^g, K_{tjz}^g, N_{tjz}^g) + \beta\left(1 - \varrho_{t+7}\right)G_{jz}^{t+1} \qquad (6)$$

Proposition 2 *A contract is a process of Pareto improvements. At least one household is better off ex-post in the contract. Two incentive constraints cannot be bind at the same time. At least one household is better off ex-ante in the contract. Two participation constraints cannot be bind at the same time. (see Appendix for proof).*

Households in the contract can gain surplus by benefiting from bequest and labor allocations. Even without any transfer or support, a bequest increases the parents' expected utility. The Pareto gain from the bequest benefit means at least one household can get a higher utility value from the contract. Because of the incentive constraints, each household's utility value is no worse than the autarky case. As one incentive constraint is binding, the other will get all the surpluses from the contract and has a higher utility level.

Proposition 3 *If the market service price is lower than that of both the parent's and the grandparent's wages, such that* $p_t^k = \min\{w_t^p, w_t^g, p_t^k\}$ *or* $p_t^n = \min\{w_t^p, w_t^g, p_t^n\}$*, then households only get care service from market, with* $n_t^p = k_t^p = k_t^g = 0$*. If the parent's wage is the lowest,, then the parent is the primary elder care (child care) provider, with* $n_t^p > 0$ *(*$k_t^p > 0$*). if the grandparent's wage is the lowest, then the grandparent is the primary child care provider, with* $k_t^g > 0$*. (see Appendix for proof)*

Wage structure and market service price determine the methods to be used to help the other household. Households choose the most inexpensive way to take care of elders or children. If the market service price is higher than that of both wages, all the care service will from the market. A household with wages higher than those of the service price uses pecuniary transfer rather than labor to help the other household. If a household provides child or elder care to help the other household, the intergenerational relationship directly reduces the household's working hour. In the contract, both households choose the most inexpensive way to provide child or elder care. If both wages are higher than those of outside service price, two households will use income transfer only to help each other. In one period, transfer and support exist at the same time. One household uses income transfer in exchange for the other household's labor support.

Proposition 4 *In an optimal contract, the expected substitution rate of marginal utility of the next period is equal to the substitution rate of the marginal utility of current period. For* $\forall\ \epsilon^g_{zt}$ *and* ϵ^p_{jt}*, there is:*

$$\frac{c^g_{tjz}}{c^p_{tjz}} = \sum_{m=1}^{J}\sum_{n=1}^{Z}\frac{\pi^p_{t+1n}\pi^g_{t+1m}}{c^p_{t+1mn}} / \sum_{m=1}^{J}\sum_{n=1}^{Z}\frac{\pi^p_{t+1n}\pi^g_{t+1m}}{c^g_{t+1mn}}$$

(See Appendix for proof)

This proposition shows the trade-off of today's and future utilities. The right term of the equations is the expected substitution rate of marginal utility of period t+1. The left term is the substitution rate of the marginal utility at time t. Two numbers are equal in the optimal contract. However, as one household has relatively low marginal utility and the other has relatively high marginal utility, the two households agree to exchange income and labor service with a relatively low marginal utility for the other's income and labor service until the substitution rates of marginal utility are equal for two periods.

Proposition 5 *If both households' income shocks are neither too large nor too small, two households will fully share the risk. Given* $\epsilon^p_{jt}(\epsilon^g_{zt})$*,* $\exists\ \epsilon^g_{kt}\ (\epsilon^p_{lt})$ *for* $\forall\ \epsilon^g_{zt} < \epsilon^g_{kt}\ (\epsilon^p_{jt} > \epsilon^p_{lt})$*, with* IC^p_t *binding, and* $\exists\ \epsilon^g_{lt}\ (\epsilon^p_{kt})$ *for* $\forall\ \epsilon^g_{zt} < \epsilon^g_{kt}\ (\epsilon^p_{jt} > \epsilon^p_{kt})$*, with* IC^g_t *binding. Between the two extreme values, both households fully share the risk and no incentive constraints are binding. (See Appendix for proof)*

The proposition defines the marginal value separating the income shocks leading to full risk sharing and partial risk sharing. When the parent's income shock is fixed, a unique marginal value exists with the parents' incentive constraint binding and the consumption ratio equaling to λ_t. If the shock is larger than the marginal value, keeping the consumption ratio equal to λ_t will cause the parents' utility value lower than the one in autarky case. When the parent's income shock is fixed, another unique marginal value exists, with the grandparent incentive constraint binding and the ex-post consumption ratio equaling to

λ_t. If the shock is smaller than the marginal value, keeping the consumption ratio equal to λ_t will cause the grandparents' utility value to be lower than that in the autarky case. In this case, the parent receives all the surplus of the contract. Both households fully share the income risk between the two extreme values. If the income shock ratio $\epsilon^g_{zt}/\epsilon^p_{jt}$ is too small, the parent's incentive constraint is binding and the grandparent receives all the surpluses. If the income shock ratio $\epsilon^g_{zt}/\epsilon^p_{jt}$ is too large, the grandparent's incentive constraint is binding and the parent get all the surplus. If the income shock ratio $\epsilon^g_{zt}/\epsilon^p_{jt}$ is neither too large nor too small, no incentive constraints bind. and two households fully share the risk.

Proposition 6 *In the no-commitment cases, a large income shock for the grandparents or a small income shock for the parents will cause a small consumption ratio* $\frac{c^g_{tjz}}{c^p_{tjz}}$. *Fixed* ϵ^p_{jt}, *if* $\epsilon^g_{mt} > \epsilon^g_{nt}$, *then* $\frac{c^g_{tjm}}{c^p_{tjm}} \geq \frac{c^g_{tjn}}{c^p_{tjn}}$. *Fixed* ϵ^g_{zt}, *if* $\epsilon^p_{kt} > \epsilon^p_{lt}$, *then* $\frac{c^g_{tkz}}{c^p_{tkz}} \leq \frac{c^g_{tlz}}{c^p_{tlz}}$.*(See Appendix for proof)*

As one side receives a small income shock, the other household helps as much as it can, until the incentive constraint binding. A small income shock will reduce the household's consumption share, if it causes the other household's incentive constraint to be binding. A small income shock ratio $\epsilon^g_{zt}/\epsilon^p_{jt}$ causes a small consumption ratio $\frac{c^g_{tjz}}{c^p_{tjz}}$. The consumption ratio determines the transfer and support intensity households give to each other. A high consumption ratio means the grandparent receives a large share of the total endowment.

Proposition 7 *In the optimal contract, a small income shock from one household cause low discount utility values and consumption of both households. If* $\epsilon^g_{mt} > \epsilon^g_{nt}$, *then*

$$g_t\left(s^g_t, s^p_t, \epsilon^g_{mt}, \epsilon^p_{jt}, G^t\right) \geq g_t\left(s^g_t, s^p_t, \epsilon^g_{nt}, \epsilon^p_{jt}, G^t\right),$$

and

$$p_t\left(s^p_t, s^g_t, \epsilon^p_{jt}, \epsilon^g_{mt}, G^t\right) \geq p_t\left(s^p_t, s^g_t, \epsilon^p_{jt}, \epsilon^g_{nt}, G^t\right).$$

If $\epsilon^p_k > \epsilon^p_l$, *then*

$$g_t\left(s^g_t, s^p_t, \epsilon^g_{zt}, \epsilon^p_{kt}, G^t\right) \geq g_t\left(s^g_t, s^p_t, \epsilon^g_{zt}, \epsilon^p_{lt}, G^t\right),$$

and

$$p_t\left(s^p_t, s^g_t, \epsilon^p_{kt}, \epsilon^g_{zt}, G^t\right) \geq p_t\left(s^p_t, s^g_t, \epsilon^p_{lt}, \epsilon^g_{zt}, G^t\right).$$

(See Appendix for proof)

Households fully share income risk in the cases with no incentive constraint binding in contracts. In these cases, a small income shock from one side causes low utility values and consumption of both households. Households partially share income risk in the cases with one incentive constraint binding. In these cases, the household with a relatively large income shock helps the household with the relatively small one as much as it can until the incentive constraint binding. A small income shock form one household causes a low utility value and consumption level of the household, and leaves the others unchanged.

5 Estimation

In this section, I discuss the identification of several key parameters of the model. The key parameters related to households' choices are wage rate w_t^i, non-working income shock υ_{it}, utility weight on leisure η, utility weight on child care α_t, utility weight on elder care weight γ_t and death rate ϱ_t. In the estimation, I assume that the working income rate and non-working income are exogenously given. I only look at each pair of grandparent and parent households. Due to the model's complexity, the arguments are mainly heuristic. Using the detailed time allocation and consumption information from my data, I identify the parameters of the preference. With information on work and income, I identify the parameters for income process based on households' characteristics. Then, I identify the parameters of value function in the following manner. I first discuss the identification on the law of motion in consumption ratio. Given the law of motion of consumption ratio across the period, the contract makes decisions in dividing the income and time into the current utility gain and savings for the future. I use an interpolation method to obtain the parameters for the value functions.

5.1 Income Process

In the data, income comes from working income and non-working income. Households make decisions on working time. I define w_t^i as females' individual working income rate. I normalize the overall time equal to 1[15]. The individual's working income I_{it} is the annual overall income from working, and it consists of income from wages, agricultural activities and business. I define the income rate $w_{it} = I_{it}/h_{it}$. The income rate follows the rules like.

$$\ln w_{it} = \Psi X_{it} + D_{year} + D_{region} + \zeta_{it}, \tag{7}$$

with X_{it} as the control variable, consisting of education level edu_{it}, age age_{it}, and age squared age_{it}^2. D_{year} are the year dummies that capture the time trend. D_{region} are region dummies that control the regional fixed effects. I use ordinary linear least squares regression to obtain Ψ, from which I can derive everyone's income rate prediction at various ages. I get the working income rate, which satisfies the following:

$$\ln w_{it} = \ln w_{i0} \times \left(\pi_1 \times age_{it} + \pi_2 \times age_{it}^2\right), \tag{8}$$

where π_1 and π_2 are from the estimated Ψ. w_{0t} is determined by education, gender, year and region.

In the non-working income part, I treat health spending as one kind of negative non-working income. Overall household health spending H_{it} is defined as overall spending in the previous year on health care service and medicine.

[15] I define the working time as h_{it} =(annual work month/12)×(daily work hour/14)×(weekly work day/7).

I_{ht} is the working income from a husband and other household members. Non-working income N_{it} is defined as the overall household income from pension, subsidy, and another non-work income source. I define net non-working income as follows:

$$P_{it} = N_{it} - H_{it} + I_{ht}. \tag{9}$$

Using the information on health spending, non-working income, and individual working income, I can obtain each household's net non-working income distribution at various ages from the data. Non-working income for the individual is as follows:

$$P_{it} = \Phi X_{it} + D_{year} + D_{region} + \varepsilon_{it}, \tag{10}$$

I use ordinary least squares regression to obtain Φ and the distribution of ε_{it}, from which I derive each age group's non-working income rate prediction at various ages.

5.2 Preference Parameters

I estimate the utility function parameters of preference, which consists of utility weight on leisure η, child care α_t, and elder care γ_t. I define the opportunity cost of time spending on child care w_{kt} or elder care w_{nt}, which is the last marginal unit of time spending on child or elder care. I normalize the child care and elder care time to 1[16]. I get the spending on child care $\pi_{it}^K = \sum_j k_t^j w_{kt}$ and elder care $\pi_{it}^N = \sum_j n_t^j w_{nt}$, j is the household member. The value of time spending is $\pi_{it}^L = w_{it}^l l_t^j$, with $w_{it}^l = w_{it}$ if i spends on working or the last unit of elder/child care is from i; $w_{it}^l = w_t^j$, if i provides child care, but the last unit is provided by j; $w_{it}^l = p_t$, if i provides child care but the last unit is provided by the market. I define household i's overall endowment[17] spending on period t is:

$$\begin{aligned} E_t^i &= R_t s_t^i + \sum_h w_{ht}^l + \epsilon_l^i - s_{t+1}^i - T_t^i \\ &\quad + \sum_h \left(n_h^i w_{nt} + k_h^i w_{kt}\right) - \sum_l \left(n_l^i w_{nt} + k_l^i w_{kt}\right) \end{aligned} \tag{11}$$

Here, h is the household member belonging to the household i, and l is the household member not belonging to the household i. The first part, $R_t s_t^i + \sum_h w_{ht}^l + \epsilon_l^i$ is the household endowment before transfer and support, s_{t+1}^i is the saving for the next period. T_t^i is the net transfer from household i to the other household. $\sum_h \left(n_h^i w_{nt} + k_h^i w_{kt}\right) - \sum_l \left(n_l^i w_{nt} + k_l^i w_{kt}\right)$ is the net child care and elder care support from household i to the other household. The overall household consumption π_{ir}^C is the household's overall spending on food, clothes, transportation, durable goods, utility, fuel, entertainment, education,

[16] For example, child care provided by j is k_t^j =(annual child care month/12)×(daily child care hour/14)×(weekly child care day/7).

[17] Endowment consists both time and money values. I get the value of the time endowment by using the overall time times the opportunity cost of the time.

beauty, and other consumption goods. Define the vector of spending $\pi_{it} = \left(\pi_{it}^C, \pi_{it}^K, \pi_{it}^N, \pi_{it}^L\right)'$ and the parameters vector $\Delta_{it} = (1, \alpha_{it}, \gamma_{it}, \eta_i)'$, I get:

$$\pi_{it} = \frac{\Delta_{it} E_t^i}{1 + \eta + \alpha_t + \gamma_t} \tag{12}$$

Define the vector of spending $y_i = \left(\pi_{it}^K, \pi_{it}^N, \pi_{it}^L\right)'$ and the parameters vector $\Lambda_{it} = (\alpha_{it}, \gamma_{it}, \eta_i)'$, I get:

$$y_i = \Lambda_{it} \pi_{ir}^C \tag{13}$$

To capture preference heterogeneity, I use a random coefficient model to estimate parameter distribution. Namely, the parameter vector Λ_{it} is specified as $\alpha_{it} = \alpha_t + \xi_{it}^{\alpha}$, $\eta_i = \eta + \xi_{it}^{\eta}$, and $\gamma_{it} = \gamma_t + \xi_{it}^{\gamma}$, where $\Lambda_t = (\alpha_t, \gamma_t, \eta)'$ is a vector of constants, and $\xi_{it'} = (\xi_{it}^{\alpha}, \xi_{it}^{\eta}, \xi_{it}^{\gamma})'$ is a vector of stationary random variables with zero means and constant variance–covariance. I use two steps generalized least squares regression (GLS) to get the preference parameters (see Appendix for details). I get

$$\tilde{\Lambda}_{it}^{g} = \left(x_{ii}^T \tilde{w}_i^{-1} x_i \right)^{-1} x_i^T \tilde{w}_i^{-1} y_i. \tag{14}$$

Here, the weight $\tilde{w}_i$ equals to the variance $\tilde{\Sigma}_t$ estimated in the first step.I then get the sample mean $\tilde{\Lambda}_t^g$ and sample variance Σ_t^g of $\tilde{\Lambda}_{it}^g$. $\tilde{\Lambda}_t^g$ captures the average utility weight in each utility term and Σ_t^g captures the heterogeneous preference distribution.

However, heterogeneity increases the state variable dimensions and causes the curse of dimensionality for the dynamic problem. If Λ_{it} is continuous, the state variable space is infinite and impossible to solve. To reduce its dimensions and simplify the problem, I assume Λ_{it} as discrete rather than continuous in the simulation. I define two types of Λ_{it} in this simplified version of heterogeneous preference. Namely, the parameter vector $\Lambda_{it} = (\alpha_{it}, \gamma_{it}, \eta_i)'$ is specified as $\alpha_{it} \in \left\{\alpha_{it}^1, \alpha_{it}^2\right\}$,$\eta_i \in \left\{\eta_{it}^1, \eta_{it}^2\right\}$ and $\gamma_{it} \in \left\{\gamma_{it}^1, \gamma_{it}^2\right\}$. The two types are specified as $\Lambda_{it}^1 = \left(\alpha_{it}^1, \eta_{it}^1, \gamma_{it}^1\right)'$ and $\Lambda_{it}^2 = \left(\alpha_{it}^2, \eta_{it}^2, \gamma_{it}^2\right)'$. I define $\bar{\Lambda}_t^i = \left(\frac{\alpha_t^i}{1+\eta^i+\alpha_t^i+\gamma_t^i}, \frac{\gamma_t^i}{1+\eta^i+\alpha_t^i+\gamma_t^i}, \frac{\eta^i}{1+\eta^i+\alpha_t^i+\gamma_t^i}\right)$, for $i \in \{1, 2, g\}$. I draw N (N=10,000) pair of η_i^g, α_{it}^g and γ_{it}^g from the parameter distribution of the estimation. I define the sample means $\mu_t^i = (\mu_{\alpha t}^i,\ \mu_{\gamma t}^i,\ \mu_{\eta t}^i)$, the sample variances $\sigma_t^i = \left(\sigma_{\alpha t}^i, \sigma_{\gamma t}^i, \sigma_{\eta t}^i\right)$ and the sample covariance $q_t^i = \left(q_{\alpha\eta t}^i, q_{\alpha\gamma t}^i, q_{\alpha\gamma t}^i\right)'$, for $\bar{\Lambda}_t^i$. The probability that i' is type 1 is ρ and that i is type 2 is $1 - \rho$. I get the moments:

$$\mu_t^g = \rho \mu_t^1 + (1 - \rho)\, \mu_t^2 \tag{15}$$

$$\sigma_t^g = \rho \left(\sigma_t^1\right)^2 + (1 - \rho) \left(\sigma_t^2\right)^2 \tag{16}$$

and

$$q^g_{\alpha\eta t} = \rho^2\sigma^1_{\alpha t}\sigma^1_{\eta t} + (1-\rho)^2\,\sigma^2_{\alpha t}\sigma^2_{\eta t} + \rho[(1-\rho)\,\sigma^1_{\alpha t}\sigma^2_{\eta t} + \sigma^2_{\alpha t}\sigma^1_{\eta t}] \qquad (17)$$

Using generalized moment method, I obtain the seven parameters (see Appendix for details). I use a two-type preference model for the following reasons. (1) The two-type model captures part of the preference heterogeneity. (2) The two-type model and continuous-type model have the average utility weight on leisure, child and elder care. (3) Adding more discrete types increases dimensions causes the calculation time and memory to grow exponentially with the dimensionality.

Discrete-type and continuous-type models have the same average spending share on leisure, child care and elder care. The time spending on child care is, for example, $\pi^K_{it} = \frac{E^i_t\alpha_t}{(1+\eta+\alpha_t+\gamma_t)w_{kt}}$, which is determined by two factors: the share of spending $\frac{\alpha_{it}}{1+\eta_i+\alpha_{it}}$, and the shadow price E^i_t/w_{kt}. The setting of preference type distribution cannot affect the average share spending on child care. I test the goodness of fit of the two-type model by comparing it with the one-type model and the continuous-type model.

First, I compare the simulation results from the two-type model with the ones from the one-type model. The one-type model spends more on child care and leisure than the two-type model. In autarky case, the average labor supply in one type is 0.25. The average labor supply in the two-type model is 0.25 in the first four periods. The difference between them is approximately 1%. The average labor supply after period 5 is 0.39 in the one-type model and 0.38 in the two-type model. In the no-commitment case, the average labor supply in the one-type model is 0.34, and that in the two-type model is 0.34 in the first four periods. The difference between them is approximately 1%. The average labor supply after period 5 is 0.33 in the one-type model and 0.34 in the two-type model. Reducing types causes the underestimation on the effects of intergenerational relationships on child and elder care, leisure and labor supply.

Second, I check the goodness of fit of the utility weight of the two-type model on the continuous type model. Therefore, I estimate the average weight on child care in the continuous distribution and in the two-type model with the estimation results. The utility weight on α_{it} satisfies a truncated lognormal distribution, and η_i also meets the truncated lognormal distribution. In this part, I use a F-test (see Appendix for details) to check the goodness of the two-type preference model. I obtain the following results: F_α is 0.69, F-value is 11,126 and p value is 0. Using the same method, I obtain the value of the F statistic on leisure $F_\eta = 0.73$, and on elder care F_γ at around 0.49-0.72. All the values have p values of less than 0.05. The two-type model fits the continuous type model well.

5.3 Substitution of Care Service between Market and Household

The real price of outside service is not the entire opportunity cost of using an outside service. In this part, I calculate the substitution rate of elder care provided by household members and outside service. The wage rate paid for

the services is not the entirety of this service cost and service quality varies among different suppliers. With the information on child and elder care, I obtain the substitution rate of elder care provided by household members and outside service. I define an outside service using dummy o_t^i - if the household uses outside child or elder care service, the number is 1; if the household uses child or elder care service from household members but not from the outside market, the number is 0. The substitution rate is ι, I parametrize the choice as:

$$o_t^i = \begin{cases} 1, \text{ if } \iota p_t \leq w_t^i \\ 0, \ otherwise \end{cases} \tag{18}$$

I use maximum likelihood estimation method to estimate the probit model (see Appendix for details).

5.4 Death Rate

The household death rate in period t is defined as that the probability that everyone dies in period t, when the household with at least one household member alive at period t-1. I use two steps to calculate the household death rate (see Appendix for details).

5.5 Parameters for Intertemporal Decisions

In this section, I estimate the form of the value functions in the dynamic problems, by using an approximation method based on simulation and interpolation. I assume each period represents four years. Grandparents retire at age 4 and parents retire at age 10.[18]

5.5.1 Autarky Case

The dynamic model is solved by backward recursion[19]. First, I draw the asset value and random shocks. I create an asset, wage and price space by drawing 5,000 grids of age varying vectors. Then for each asset level, I use the Gauss-Hermite quadrature method[20] to draw 10 quadrature nodes of income shocks $(\epsilon_{1t}, ...\epsilon_{10t})$ from the estimated distribution from Equation 10. These nodes are chosen by dividing the support of the normal distribution into 10 equiprobable intervals and then finding the conditional means within each interval. The quadrature nodes $(\epsilon_{1t}, ...\epsilon_{10t})$, lie in the domain of normal distribution, and the quadrature weights $(W_{1t}, ...W_{10t})$ are assigned appropriately to the approximate of the expected value.

As the problems for the subsequent periods are fundamentally the same, I discuss only the problem in period t. The basic logic is as follows: I use

[18] According to the Survey on Fertility and Birth Control in China the average age at which females have their first child had increased from 22 in 1991 to 28 in 2010. In the model, therefore, grandparents are 24 years older than parents. Parents' fertility age is 28 on average.

[19] This is the standard approach in this literature. For example, see Keane & Wolpin (1997).

[20] This method follows the Gauss-Hermite rule in Chapter 7 of Judd (1998).

the estimated expected value function from the previous period to set up the agent's objective function to solve the problem. After obtaining the solutions, I calculate the autarky value for each asset draw. Using these coefficients, I construct the estimated expected value function which is used to solve the t-1 period problem. Then I solve the problems in the same fashion backward until t=1. The idea is stated formally as laid out below.

Suppose I have already solved for the emax function for age t+1 and the functional form of the value function $V_{t+1}^{aut}(s_{t+1}^p)$ is already solved. Given the quadrature nodes, I calculate $v_t^{aut}(s_t^p, \epsilon_{kt})$ with respect to s_t^p and ϵ_{kt}. Furthermore, in integrating for each value of the shock vector, I get the optimal consumption, labor supply, elder care and child care to derive $v_t^{aut}(s_t^p, \epsilon_{kt})$. The autarky value is given by:

$$V_t^{aut}(s_t^p) = \sum_{k=1}^{10} v_t^{aut}(s_t^p, \epsilon_{kt}) W_{kt} \tag{19}$$

By solving each asset draw, I get the relationship between $V_t^{aut}(s_t^p)$ and s_t^p. Then I use a linear model to approximate the expected value function:

$$\bar{V}_t^{aut}(s_t^p) = \varpi_0^t + \varpi_1^t \log s_t^p + \varpi_2^t \log p + \varpi_3^t \log w_t^i + \xi_t \tag{20}$$

Using linear regression, I obtain the coefficients and the fit value. Using backward induction, I solve the optimization problem for every period. After solving the dynamic problem, I obtain a sequence of coefficients for value functions. I use such a simple regression form for two reasons: (1) This simple form captures the fact that the value function is concave; (2) There is a trade-off in choosing the form of regression. On one hand, making the regression more complex could possibly improve the predictable power of the regression; on the other hand, the complex form of regression would make the first-order conditions for t-1 period problem quite complex, which is very difficult to solve.

5.5.2 No-commitment Case

In the no-commitment case, asset levels s_t^g and s_t^p, and consumption ratio λ_t (G^t)[21], are the state variables in period t. A contract places the utility weights $\frac{1}{\lambda_t+1}$ on the parent's utility and $\frac{\lambda_t}{\lambda_t+1}$ on the grandparent's utility before the realization of income shocks. After the income shocks, the contract allocates income and labor service according to the ex-post utility weight, which is determined by the ex-ante utility weights and incentive constraints. In each state, the contract allocates total endowment according to the ex-post weights.

I follow the backward recursion method to estimate the value function. Since the problem for each period is fundamentally the same, I only discuss the problem in period t and assume that households know the form of the value functions in period t+1. In period t, I create an asset space by drawing 5,000 pairs of age

[21] λ_t defines the ex-ante share of the endowment of each household gets. Unique λ_t exists for each G^t, given s_t^g and s_t^p,

overall varying assets $s_t^g + s_t^p$, consumption ratio λ_t, and wage rate. Then, for each draw, I use the Gauss-Hermite quadrature method to draw 10 quadrature nodes for the parent $(\epsilon_{1t}^p, ...\epsilon_{10t}^p)$ and 10 quadrature nodes for the grandparent $(\epsilon_{1t}^g, ...\epsilon_{10t}^g)$. For each income shock combination of ϵ_{lt}^p and ϵ_{mt}^g, the weight is $W_{lt}W_{mt}$. The basic logic is as follows: in step 1, I match s_t^g/s_t^p with $s_t^g + s_t^p$ and λ_t; in step 2, I set up the agent's objective function and obtain the optimal choices at each state; in step 3, I calculate the value function of the agent for each asset draw, and identify the form of value functions. The second and third steps follow the same estimation method in the autarky case.

The first step reduces the number of state variables from three to two by matching s_t^g/s_t^p, with $s_t^g + s_t^p$ and λ_t. In proposition 4, the ex-post marginal utility ratio of period t is equal to the ex-ante marginal utility ratio in period t+1, such that:

$$c_{tjz}^g/c_{tjz}^p = E\left(1/c_{t+1}^p\right)/E\left(1/c_{t+1}^g\right) \tag{21}$$

and

$$\lambda_t = E\left(1/c_{t+1}^p\right)/E\left(1/c_{t+1}^g\right) \tag{22}$$

which define the law of motion of the consumption ratio. A unique s_t^g/s_t^p exists for each pair of $s_t^g + s_t^p$ and λ_t. Using the two equations and incentive constraints, I can identify the ex-post utility weight in each state, and match s_t^g/s_t^p with $s_t^g + s_t^p$ and λ_t. The state variables of the contract become $s_t^g + s_t^p$ and λ_t. I then define the ex-post consumption ratio on the grandparent κ_{jzt}, so that

$$\kappa_{jzt} = \frac{\partial P^{t+1}(s_{t+1jz}^p, s_{t+1jz}^g, G_{jz}^{t+1})}{\partial G_{jz}^{t+1}}, \tag{23}$$

and

$$\kappa_{jzt} = c_{tjz}^g/c_{tjz}^p, \tag{24}$$

with income shocks ϵ_{kt}^p and ϵ_{kt}^g. In the optimal contract with s_t^g and s_t^p, a lower bound $\underline{\kappa}_{jzt}$ exists, with the grandparent's incentive constraint binding; and a higher bound $\bar{\kappa}_{jzt}$ exists, with the parent's incentive constraint binding. Between the two bounds, $\kappa_{jzt} = \lambda_t$. $\underline{\kappa}_{jzt}$ is strictly increasing and $\bar{\kappa}_{jzt}$ is strictly decreasing on s_t^g/s_t^p. Since κ_{jzt} is a function of s_t^g/s_t^p, I get:

$$m\left(s_t^g/s_t^p\right) = \sum_{l=1}^{10}\sum_{m=1}^{10}\left(W_{lt}W_{mt}\kappa_{jzt}/c_{tjz}^g\right)/\sum_{l=1}^{10}\sum_{m=1}^{10}\left(W_{lt}W_{mt}/c_{tjz}^g\right) - \lambda_t \tag{25}$$

Using the moment $m\left(s_t^g/s_t^p\right) = 0$, I use GMM method to find the unique s_t^g/s_t^p for each pair of $s_t^g + s_t^p$ and λ_t.

The second step is to solve the agent's objective function for each pair of ϵ_{kt}^p and ϵ_{kt}^g. First, without taking incentive constraints into consideration, I set the ex-post consumption ratio $\tilde{\kappa}_{jzt} = \lambda_t$. I use $\tilde{\kappa}_{jzt}$ to get the utility value weight on each household and use a first order approach to solve the problem.

With the solution, I get $\tilde{g}_t(s_t^p, s_t^g, \epsilon_{kt}^p, \epsilon_{kt}^g)$ and $\tilde{p}_t(s_t^p, s_t^g, \epsilon_{kt}^p, \epsilon_{kt}^g)$. I then take the incentive constraints into consideration. If $\tilde{g}_t(s_t^p, s_t^g, \epsilon_{kt}^p, \epsilon_{kt}^g) < v_{gt}^{aut}(s_t^g, \epsilon_{kt}^g)$, I set the grandparent's incentive constraint binding, and the parent gets all the Pareto gain. If $\tilde{p}_t(s_t^p, s_t^g, \epsilon_{kt}^p, \epsilon_{kt}^g) < v_{pt}^{aut}(s_t^p, \epsilon_{kt}^p)$, I set the parent's incentive constraint binding, and the grandparent gets all the Pareto gain. Otherwise, $g_t(s_t^p, s_t^g, \epsilon_{kt}^p, \epsilon_{kt}^g) = \tilde{g}_t(s_t^p, s_t^g, \epsilon_{kt}^p, \epsilon_{kt}^g)$, and $p_t(s_t^p, s_t^g, \epsilon_{kt}^p, \epsilon_{kt}^g) = \tilde{p}_t(s_t^p, s_t^g, \epsilon_{kt}^p, \epsilon_{kt}^g)$. In this step, I get $g_t(s_t^g, s_t^p, \lambda_t, \epsilon_z^g, \epsilon_j^p)$ and $p_t(s_t^g, s_t^p, \lambda_t, \epsilon_z^g, \epsilon_j^p)$ for each pair of ϵ_{kt}^p and ϵ_{kt}^g.

The third step is to calculate the ex-ante value functions for each pair of assets and consumption ratio, and get the function form. With the $g_t(s_t^g, s_t^p, \lambda_t, \epsilon_z^g, \epsilon_j^p)$ and $p_t(s_t^g, s_t^p, \lambda_t, \epsilon_z^g, \epsilon_j^p)$ from solved from the previous step, the value functions for each asset and consumption ratio draw is the following:

$$G^t\left(s_t^g, s_t^p, \lambda_t\right) = \sum_{l=1}^{10}\sum_{m=1}^{10}\left[g_t(s_t^g, s_t^p, \epsilon_z^g, \epsilon_j^p)W_{lt}W_{mt}\right], \tag{26}$$

and

$$P^t\left(s_t^g, s_t^p, \lambda_t\right) = \sum_{l=1}^{10}\sum_{m=1}^{10}\left[p_t(s_t^g, s_t^p, \epsilon_z^g, \epsilon_j^p)W_{lt}W_{mt}\right]. \tag{27}$$

I get the expected value functions for each pair of $s_t^g + s_t^p$ and λ_t. By solving each asset draw, I get the relationship between value functions and state variables, given the income shock distribution. I then use a log linear function to approximate the expected value function:

$$\begin{aligned} G^t\left(s_t^g, s_t^p, \lambda_t\right) &= \varphi_0^t + \varphi_1^t \log(s_t^g + s_t^p) + \varphi_2^t \log \lambda_t \\ &\quad + \varphi_3^t \log p + \varphi_4^t \log w_t^p + \varphi_5^t \log w_t^g + \xi_t^g \end{aligned} \tag{28}$$

and

$$\begin{aligned} P^t\left(s_t^g, s_t^p, \lambda_t\right) &= \phi_0^t + \phi_1^t \log(s_t^g + s_t^p) + \phi_2^t \log \lambda_t \\ &\quad + \phi_3^t \log p + \phi_4^t \log w_t^p + \phi_5^t \log w_t^g + \xi_t^p \end{aligned} \tag{29}$$

Using regression, I get the coefficients and fit values. Starting at t=20, I use backward recursion to solve the problems.

5.5.3 Full-commitment Case

The full-commitment case chooses a constant utility weight on each household in every state. The estimation of a full-commitment case model is the same as the estimation of a no-commitment case model, but the consumption ratio is fixed. For each period, I create an asset space by drawing 5,000 pairs of age overall varying asset $s_t^g + s_t^p$, consumption ratio λ_t, wages, and price, and estimate the form of value functions.

6 Data

6.1 Data and Sample Selection

I use data from the China Health and Nutrition Survey[22] (CHNS) and the China Health and Retirement Longitudinal Study (CHARLS)[23] to perform structural estimation.

The CHNS is a longitudinal survey project collected 11 waves since 1989. The survey took place over a 3-day period using a multistage, random cluster process to draw a sample of about 4,400 households with a total of 26,000 individuals in nine provinces that vary substantially in geography, economic development, public resources, and health indicators. In addition, detailed community data were collected through surveys on food markets, health facilities, and other social services. A multistage-random cluster process was used to draw the samples surveyed in each of the provinces. Counties in the 15 provinces[24] were stratified by income (low, middle, and high), and a weighted sampling scheme was used to randomly select 4 counties in each province. A provincial capital and a lower-income city were also selected when feasible. CHNS tracks the households that in previous samples. In the summary statistics, I divided the whole sample into 2 periods. Group 1 is the 2000-2004 sample and group 2 is the 2006-2011 sample. I use group 2 in my estimation. I restrict the analysis to females who are at least 20 years old. I use the CHNS data to identify the utility weight on child care and leisure, wage rate, and the non-working income distribution.

CHARLS is a biennial survey that aims to represent of the residents of China aged 45 and older, with no upper age limit. The national baseline sample size is 10,287 households and 17,708 individuals, covering 150 counties in 28 provinces. The baseline of the CHARLS pilot took place in two provinces in fall of 2008. The first national baseline wave was fielded from 2011 to 2012. Wave 2 was fielded in 2013. The household survey includes demographic background, household information, health status and functioning, health care and insurance, work information, household and individual income, expenditure and assets. I use CHARLS data to identify the utility weight on elder care, health spending distribution, and age patterns of transfer and support.

The death rate data are from the National Population and Reproductive Health Science Data Center[25]. I use the data of 2005 to get the death rate for

[22] CHNS data is an international collaborative project between the Carolina Population Center at the University of North Carolina at Chapel Hill and the National Institute for Nutrition and Health at the Chinese Center for Disease Control and Prevention. The official website of CHNS is: http://www.cpc.unc.edu/projects/china

[23] CHARLS is a project of China Center for Economic Research. The data is based on the Health and Retirement Study and related aging surveys such as the English Longitudinal Study of Aging and the Survey of Health, Aging and Retirement in Europe. The official website of CHARLS is: http://charls.ccer.edu.cn/en

[24] The provinces are Beijing, Chongqing, Guangxi, Guizhou, Heilongjiang, Henan, Hubei, Hunan, Jiangsu, Liaoning, Shaanxi, Shandong, Shanghai, Yunnan, and Zhejiang.

[25] The National Population and Reproductive Health Science Data Center calculated death rate by using the data from the National Census of Population in 1982, 1990, 2000, and 2010.

the data before 2008 and the data of 2010 to get the death rate of the years following 2008 (see Appendix for details)..

6.2 Descriptive Statistics

Table 1 describes the working and income information. The data on female working choices indicates that, between 2000 and 2011, the average working rate decreased from 72% to 62%. Figure 1 reports the average female working time by age for various periods. The result indicates that females in urban areas decrease their working time before they reach the age of 50, which is much earlier than the legal retirement age. These households in the CHNS data are getting older and give a higher weight to the older population, which reduces the average employment rate. The first graph in Figure 2 shows the income rate distribution by age cohort. The development of education system enables the younger generations to receive better education than their parents. The younger generations are more productive and less likely to work in the agricultural sector. Therefore, they have higher incomes than the older generations. In the sample, 7,950 females between the age of 20 and 70 have both working time and working income information. Table 1 shows that the average working time is 0.33 and the average female income rate is about 50,000 yuan.

Table 1 contains summary statistics on transfer, child care and elder care (see Table 7 in the Appendix for details). About 15% of people older than 60 years old age take care of their grandchildren during the survey period. Only 19% of households use outside child care service. Figure 3 reports the change in transfer and support change by the grandparent's age. Both support and transfer from parent to grandparent increase as the grandparent gets older. Grandparents take care of their grandchildren before the age of 60 and are being taken care of by parents after age 70. The net value of transfer at age 45 is 0, and it increases to about 12,000 yuan at age 90.

Table 1 also describes the household characteristics (see Table 8 in the Appendix for details). For most households, the most important asset is house. The average asset[26] level is 423,270 Yuan in CHARLS and 455,223 Yuan in CHNS. The average consumption level is 30,316 Yuan in CHARLS and 31,774 Yuan in CHNS. CHARLS have more information about asset level and consumption[27]. Therefore, the consumption level in CHARLS is higher than that in CHNS. I use the mean and variance of the net non-working income[28] distribution in CHNS data to draw the net nonworking income in simulation. Using the information on non-working income, working income and health spending, I calculate the household net non-working income of 27,495 yuan on average.

The center calculated death rate the average death rate by age for each gender in China. Data retrieve from: http://www.poprk.org/metadata/detail/254

[26] Figure 4 in the Appendix reports asset levels by age. In the data, the asset level reaches its highest point at age 55.

[27] CHNS does not have information about financial assets and household spending on clothes and transportation. CHARLS has all the information.

[28] Table 2 in the Appendix reports the distribution of net non-working income of various age groups.

6.3 Parameter Values

This section reports the estimation results. The initial female working income rate is given in Table 2. In period 1, the average parent's wage rate is 55,457 yuan and the average grandparent's wage is 59,519 yuan. Grandparent's wage is higher than that of parents on average. After the second period, As wages grow, the average wage of parents is higher than that of grandparents. To draw the wage rate in the simulation, I estimate the correlation between each pair of wage rates w_{p0} and w_{g0} by matching the households of the parent and grandparent in the CHNS data. The correlation I find between w_{p0} and w_{g0} is 0.46. I use the wage growth rate from 2006 to 2011 to obtain the predicted wage rate in the benchmark simulation. From the regression of the working income rate (see Table 9 in the Appendix for details), the coefficient on age is 0.056; the coefficient on age square is -0.0007. The highest working income rate among all age cohorts in this period is age 40. I use the wage growth rate from 2000 to 2004 as the second wage structure to verify to the effect of the wage structure on labor supply. The coefficient on age is 0.082 and the coefficient on age square is -0.0008. The highest working income rate among all age cohorts in this period is age 50.

Table 2 presents the main estimation results of the preference parameters. The first set of columns shows the results of the random coefficient linear regression model specifications. The first and second columns are the mean and variance of the vector of constants Λ_t, and the third column is the vector of stationary random variables ξ_{it}. Table 2 shows that the utility weights on elder care increase by age. γ_t is 0.06 before 72, increases to 0.20 from 72 to 80, and is 0.52 after 85. The utility weight on child care is 0.31 and that on leisure is 0.88. The second and third set of columns shows two types of preference results from linear regression model specifications. Type 1 households have less weight in leisure, but more weight in child care and elder care than type 2. The estimation result shows that 42% of households are type 1 and 58% are type 2. So, in both no-commitment and full-commitment cases, 34% of parents and grandparents are type 1; 18% are both type 2; 24% have type 1 grandparents and type 2 parents; and 24% have type 1 parents and type 2 grandparents. Table 2 also reports that the substitution rate between outside child care service and household child service is 2.5, and the substitution rate between outside elder care service and household elder service is 1.9. I assume that the discount factor β is 0.97, which corresponds to a rate of time preference of 3% per year. I assume that the real interest rate R is 1.01, which corresponds to the average interest rate in China from 2006 to 2013 (World Bank 2016).

7 Life-Cycle Fit

In this part, I use simulation to predict households' decisions on savings and labor supply along the life-cycle. The parameters have been presented in the previous section. The simulation focuses on household choices within the inter-

generational relationship. So, in the section and policy analysis section, I only consider the household behavior within the contract. The estimated results indicate the average number of the entire sample.

7.1 Simulation Procedure

To create the simulation sample, I draw a random sample of 5,000 pairs of savings according to the parametrization described above. Using the simulation results, I offer some economic intuition related to the life-cycle profiles of household labor supply, transfer, support and savings from the model. From the CHNS data, I draw the initial wage rate w_0^p of parents using the wage rate distribution of females ages 20 to 23 and draw the initial wage rate w_0^g of grandparents using the wage rate distribution of females ages from 41 to 45. The correlation between w_0^g and w_0^p is 0.46. Using the parameters from the wage growth equation, I obtain individual i's predicted wage rate w_t^i at time t. The average real child care and elder care market service cost is 62,500 yuan[29]. The non-working income is drawn according to the distribution of the net non-working income[30]. From the CHNS data, I draw parents' initial asset level s_0^p using the asset distribution of females ages 20 to 23 and draw grandparents' initial asset level s_0^g using the asset distribution of females ages from 41 to 45. In period 0, the average initial asset level of grandparents is 375,000 yuan, while the average initial asset level of parents is 75,000 yuan. The correlation between s_0^g and s_0^p is 0.53. For everyone in the sample, the simulation uses: three fixed individual characteristics (working income rate, non-working income distribution, and care service price), three initial state variables (parents' savings, grandparents' savings and consumption ratio).

The initial consumption ratio is defined by assuming both households divide the ex-ante Pareto gains equally before they join the contract:

$$\lambda_1 = \arg\max \left[G^1 - V_1^{aut}(s_1^p)\right]^{\frac{1}{2}} \left[P^1\left(s_1^g, s_1^p, G^1\right) - V_1^{aut}(s_1^g)\right]^{\frac{1}{2}} \tag{30}$$

In each period, I draw the income shock from the net non-working income distribution of each pair of households. I then solve the optimal contract, given the consumption ratio and the savings. Next, I find out the consumption share in period t according Equations 23 and 24. Using Equation 11, I get:

$$s_{t+1}^g + s_{t+1}^p = \frac{\beta A_1 \left(E_t^p + E_t^g\right)}{\kappa_{jzt} A_2 + A_3} \tag{31}$$

Here $\beta A_1 = \beta\left[\left(1-\rho_{t+7}\right)\left(\kappa_{jzt}\varphi_1^t + \phi_1^t\right) + \rho_{t+7}^t \varpi_1^t\right]$ is the weight on overall saving; $\kappa_{jzt} A_2 = \kappa_{jzt}\left(1 + \alpha_{t+6} + \eta_{t+6} + \gamma_{t+6}\right)$ is the weight on the grandparents'

[29] By calculation, I get the average child care price is 24,889 Yuan, and the average elder care price is 31,903 Yuan. The substitute rate between outside child care service and household child service is 2.5, and the substitution rate between outside elder care service and household elder service is 1.9. I assume both prices are 62,500 yuan.

[30] The distribution of net non-working income by age is shown in Table 2 in the Appendix.

utility level; $A_3 = 1+\alpha_t+\eta_t+\gamma_t$ is the weight on parent's utility level. By Equations 12 and 13, I can get the spending on each term. By matching s^g_{t+1}/s^p_{t+1} with $s^g_{t+1}+s^p_{t+1}$ and κ_{jzt},according to Equations 21 and 22, I get s^g_{t+1} and s^p_{t+1}, and go to next period. Using the same method, I get the predicted values from period 1 to 20.

7.2 No-commitment Case Fits Data Better

The no-commitment case model fits the data better than the full-commitment case and the autarky case. To illustrate this point, I compare the actual average working time, savings, child care, and elder care choices from the data predicted by the no-commitment model. Table 3 presents the actual and predicted values of the no-commitment case on labor and savings as well as other measures. The dynamic model reasonably predicts the working, leisure, transfer and saving choices. The chi-square goodness of fit tests does not reject the null hypothesis that these values are different.

The no-commitment case fits the data on labor supply. The autarky case fails to explain the age pattern of female supply. The decline in the female labor supply before retirement age does not occur in the autarky case. Figure 1 shows that the female working time of the rural[31] population aged 40-45 decreases from about from 0.4 to 0.1, which is about 5 years earlier than the legal retirement age. Figure 5 displays the simulation results on labor supply[32]. In the autarky case, the labor supply of grandparents decreases smoothly until the legal retirement age. In both the no-commitment and full-commitment cases, grandparents work less to provide more care for their grandchildren from period 1 to period 4[33], and parents work less to provide more elder care from period 5 to period 10[34]. The combination of the two effects causes the decline in labor supply before retirement age in the data. Labor support through intergenerational relationships changes the labor supply of both parents and grandparents. In addition, in the autarky case, parents provide child care and work less. The scenario is not shown in the data and the no-commitment case, because grandparents take care of grandchildren in both settings. Without intergenerational relationships, the autarky case cannot explain the age patterns of female labor supply. The no-commitment case under predicts the effect of intergenerational relationships on the labor supply before retirement age. The second graph in Figure 5 compares the age patterns of labor supply from the simulation and

[31] I define rural households and urban households by the households' Hukou. A Hukou is resident recording systems required by law in China. Hukou officially identifies a person as a resident of an area, and rural or urban resident.

[32] Table 3 in the Appendix shows the simulated value of the labor supply by age.

[33] Figure 1 in the Appendix compares the female labor supply of the households with and without children. In this part, I define the female without children, as a household that does not have children and whose households' members were at least 35 years old before 2004. 1,465 females meet this standard and 9,415 females have at least one child.

[34] Figure 2 of Appendix compares the female labor supply of the households with and without parents alive. In this part, I define the female without a parent as a household that does not have parents or parents-in-law. 1,565 females meet this standard, while 6,923 females have at least one living parent or parent-in-law.

data. In reality, parents also need to take care of their own grandchildren. My model ignores the problems on grandchild caring on the part of parents. Without the grandchild care problem, the labor supply of parents from simulation is larger than that in the data at around age 50.

The no-commitment case fits the data on saving. The full-commitment case cannot make predictions on the savings of a single household. In the full-commitment case, which is free of incentive problems, only the overall savings and consumption ratio matter in the allocation of income and labor. Without a bequest tax, the saving allocation of the two households has no effects on working, child care, and elder care decisions in the full-commitment case. In the no-commitment case, a single household's saving determines the outside option of the contract and affect the endowment allocation between households; a unique savings-share rate exists in each state. Without a single household's saving information, I can only identify the labor supply, child care, and elder care decisions in full-commitment contract. The no-commitment case model predicts saving[35] better at the later stages of the relationship. The second graph of Figure 6 compares the age patterns of saving from simulation and data. The simulation results of the no-commitment case fit the data well in the later stages of the relationship. In the data, some parents cohabitate with the grandparents and thus their assets cannot be separated. Therefore, parents are given a high average asset level during the earlier periods of the relationship.

7.3 Choices of Transfer and Support across Periods

Households' savings and wage rates determines the choices and intensity of support and transfer. Figure 7 illustrates the simulated net transfer and support value[36] by age in the no-commitment case. At the early stages, parents are less motivated to use transfer exchange bequests, because the grandparents' death rate is low. Without a bequest incentive in the first three periods, the net transfer and support values are only above zero. But as the grandparents grow older and are more likely to die, bequests are more likely to occur. During the first three periods, parents pay transfers to grandparents' child care. After period 4, bequest incentives are mainly changed by two forces: the grandparents' death rate, which increases by age, and asset level, which decreases by age after period 4. From period 4 to period 10, the first force dominates the second force. The bequest incentive increases and the net transfer value grows from period 4 to period 9. After period 10, the second force dominates the first force, and the net transfers and support value decreasing by age. The age patterns of endowment composition fit the actual data well, as showed in Figure 3. The endowment composition determines the intensity of support and transfer.

Figure 7 also indicates the composition change of transfer and support by age. The bequest incentives and coinsurance affect the overall net transfer and support value in the contracts. The composition of transfer and support

[35] Figure 4 in the Appendix shows the simulated value of saving by age.

[36] The net transfer and care support value are defined by net transfer plus labor support value. Labor support value is defined by the support hour time support opportunity cost.

changes across the periods. The wage structure determines whether the household chooses support or transfer to help one another. At the early stage of the relationship, because grandparents' wage rates are lower than those of parents and the outside care service rates, grandparents become the primary child care providers. Parents use transfer to pay for the support. After period 5, when grandparents need elder care and parents' wage is lower than the service price, parents use an outside elder care service and transfer in exchange for future bequests. After period 10, when parents are no longer able to provide elder care and work, they only use transfers in exchange for bequests. The age patterns of the intensity of transfer and support fit the actual data (see Figure 3).

7.4 Effect of Wage Structure on Labor Supply

Wage structure affects labor supply by determining the choices of child and elder (see Table 10 in the Appendix for details). Figure 5 describes the simulated results of labor supply of the no-commitment, autarky, and full-commitment cases. Given the wage structure and service price, in the autarky case, parents provide child care by themselves and grandparents acquire elder care service from the market. In the no-commitment and full-commitment cases, parents use grandparents' child care service and grandparents use parents' elder care service. Figure 5 presents the difference in the labor supply between the full-commitment and no-commitment cases. In the full-commitment case, households provide more assistance to each other. Grandparents provide more child care and parents give more elder care in the full commitment case than the no-commitment case. In the full-commitment contract, the labor supply of parents is about 5% higher before period 4 and 5% lower after period 5 than that in the no-commitment case. Grandparents substitute the parents for child care in the contract, parents' labor supply increases by 26%, and grandparent' labor supply decreases by 19% in the first 4 periods, compared to the autarky case. However, when grandparents grow older, parents' labor supply is 13% less than that in the autarky case.

The wage structures influence labor supply by affecting the households' elder care and child care choices. To help understand how the wage structure changes the labor supply behavior, I add a new wage structure, which is presented in the second graph in Figure 2. In the new wage structure, parents become the primary child care providers (details see Table 10 in the Appendix). In the contracts, parents' labor supply does not vary significantly compared with that in autarky case in the first 3 periods. The wage structure determines who the primary care providers are, which affects whether or not the substitution affects on labor supply exist in intergenerational relationships. In a fast-growing economy, such as China and India, younger generations have higher wages rate than the older ones. In these countries, intergenerational relationships have greater effects on the labor supply than other countries.

7.5 Richer Gets More Transfer and Support

Richer households have more resources for exchange in intergenerational relationships than poor households. Grandparents use bequests in exchange for parents' income and labor service in contracts. Bequest motives encourage grandparents to save more in the contracts than in the autarky cases. Figure 6 describes the simulated results of savings. Grandparents save more in the no-commitment case than in the autarky case. Besides, grandparents obtain a higher utility level of securing less expensive elder care, greater income and labor service from the intergenerational relationship, and therefore greater saving more money to save. Grandparents save, on average, 19% more in the no-commitment case than the autarky case (see Table 10 in the Appendix for details). Bequests discourage parents to save in the no-commitment case. Parents save 6% less on average in the no-commitment case than in the autarky case. In the no-commitment case, when grandparents die, bequests increase parents' savings in future. Bequests reduce the marginal utility of savings, and reduce parent's motivation to save. Bequests increase grandparents' but decrease parents' savings.

To confirm the effect of the incentive problem on household decisions, I conduct two experiments. In the first experiment, I change the average initial asset level of grandparents from 375,000 to 125,000 and leave parents' initial saving unchanged. Grandparents have small bequest in exchange for parents' income and elder care service. Figure 8 presents the change in transfer and labor supply brought about by the adjustment. The first figure shows the new predicted values of transfer and support. The average net transfer and support value decreases from around 15,000 to around 5,000. The transfer value is negative in some periods. Grandparents use both bequest and transfer in exchange for parents' elder care support, as bequests are not enough to meet the demand for elder care. Parents works more, when the grandparents have only a small savings. The second graph in Figure 8 indicates the effect of the asset change on labor supply. In the first 4 periods, grandparents provide more child care than the benchmark setting. Parents receive more child care from grandparents, thus increasing their working time. After period 4, the bequest incentive is weakened by low level asset level. Parent's working time increases by about 10%. In the second experiment, I leave the initial assets and other parameters unchanged. I select a pair of grandparent and parent by drawing the initial assets and wage rates level from the distributions. I set a fixed income and asset path before period 5. I assume 2 cases in period 5. In the no-commitment case, grandparents with good income shock obtains 13% greater elder care service from parents than those with a bad income shock. The good income shock increases the labor supply of parents by 7%. In the full-commitment case, the grandparent with good income shock gets 3% less elder care service from parent than those with bad income shock. The good income shock decreases the labor supply of parents by 2%.

The motive source affects households' transfer and support behavior in intergenerational relationships. In the no-commitment cases, coinsurance and be-

quest affect the amount of support and transfer that households provide to one another. These factors establish the intensity of mutual aid and the size of the effects on labor supply. Households transfer money and support one another in exchange for current or future income, and labor service of others. In poor economic conditions, households with smaller endowments exchange transfer and support from other households. Thus households with fewer assets obtain less support and transfer in the no-commitment contracts. The result is opposite to the prediction of the standard altruistic models, in which poorer households obtain more help in intergenerational relationships[37]. In the altruistic models, households help others to increase their own utility level by increasing other household's utility. Households with fewer assets obtain more help from the others. The differences in the motives for transfer and support cause the differing predictions of the two models.

7.6 Heterogeneous Responses

Intergenerational relationships have a greater influence on households with more demand on care service than on those with less. Figure 9 shows the changes in labor supply change caused by intergenerational relationships between type 1×1 households (in which both households are type 1) and 2×2 households (in which both households are type 2). Intergenerational relationships have a more significant effect on the labor supply of type 1×1 households, because type 1 households need more child and elder care than type 2 households (see Table 10 in the Appendix for details). In periods 1 to 4, intergenerational relationships increase the labor supply of type 1×1 parents by 40%, increase that of type 2×2 parents by 2%, and decrease that of type 1×1 grandparents by 7%, and increases that of type 2×2 grandparents by 19% .

The demand for child care and elder care influences the effect of intergenerational relationships on labor supply. Intergenerational relationships have great impact on the labor supply of households with a high utility weight on child care or elder care.

8 Policy Analysis

To illustrate how household labor supply, savings, child care, elder care and transfer rates change as the public policies, I calculate the percentage changes in these measures as I impose the policies. The settings of death rate, income process, and wage structures are the same as the benchmark settings in the previous section.

[37] Parents account both for the individual and relative economic position of their children and give them transfers or sharing of inheritance to their children unequally (Schanzenbach & Sitkoff 2008).

8.1 Child Care and Elder Care Subsidies

This section shows the effect of child care and elder care service subsidies on labor supply decisions. Keeping other parameters and settings constant, I compare the choices of households facing zero subsidy, 10% subsidy, and 20% subsidy on elder or baby care service purchased from the market and a 20% subsidy on both elder and child care service (see Table 11 in the Appendix for details).

The 20% care service subsidy has a much greater impact on labor supply than the 10% care service subsidy. Figure 10 illustrates the simulated effects of the subsidy on both kinds of care services. With a 10% care subsidy, the service price remains higher than most of females' wages. However, with a 20% subsidy, most parents' and grandparents' wages are higher than the price of outside service. The 10% care subsidy increases parents' and grandparents' labor supply by only 6%. The 20% care subsidy increases the grandparents' labor supply by 41%. Child care subsidy affects grandparents' labor supply in the earlier stages. Elder care subsidy affects labor supply at the later stages of the contract. The 20% child care service subsidy increases the labor supply of younger females by 24% in the autarky case and by 10% in the no-commitment case. The subsidy increases the labor supply of older females by 39% from period 1 to period 3. The 20% elder care service subsidy increases the labor supply of younger females by 13% in the no-commitment case after period 4.

Child care and elder care subsidies affect the labor supply of both households. When grandmothers are the primary child care providers, the low-cost market service can substitute for grandmothers in the provision of child care. The substitution increases the labor supply of old females. An elder care subsidy can affect the labor supply of parents. When the mothers are the primary elder care providers, the low-cost market service substitute for the mothers in the provision of elder care. Moreover, the labor supply of young females also increase. Child care and elder care service subsidies reduce the demand for support through intergenerational relationships by encouraging households to utilize the formal care service. The subsidies increase the females' labor supply, when the subsidized service price is lower than the care providers' wages.

8.2 Delay Mandatory Retirement Age

This section presents the effects of delaying the mandatory retirement age on labor supply and saving decisions of households. Keeping other settings unchanged, I delay the retirement age from age 9 to 10. The mandatory retirement ages of females are between 55 and 60 years old in China (see Figure 7 in the Appendix for the details). In 2015, China's Ministry of Human Resources and Social Security announced a new retirement plan to delay the mandatory retirement age to 65 years old in the 2020s [38]. Therefor, female labor supply

[38] The Chinese government plans to take pressure off the nation's increasingly strained pension system by gradually raising retirement ages for the nation's millions of workers between 2017 and 2022. The nation's Ministry of Human Resources and Social Security has declared that eligibility ages for men, women, urban workers and farmers will be raised in steps by

is predicted to change because of the new retirement plan (see Table 11 in the Appendix for details).

With the benchmark wage structure, retirement delay causes a insignificant effect on labor supply. In this part, the working income rate follows the growth rate of age as described in Table 4. Figure 11 shows the simulated effects on retirement delay. In the benchmark wage structure, grandparents' wage is lower than parents' wage and the service price. Before and after the policy is changed, grandmothers are always the primary child care providers. Delaying the retirement age has limited effects on the baby care choice, as well as the labor supply of grandparents and parents. The policy reduces the parents' labor only by 8% with the benchmark wage structure.

As I narrow the wage gap between parents and grandparents, the retirement delay causes a large effect on labor supply. The wage structure[39] has a small wage gap between young and old females. The retirement delay has different effects on the new wage setting. Delaying the mandatory retirement age remove grandmother from child care. The retirement delay decreases parents' labor supply and increases grandparents' labor supply in period 4. Delaying the retirement age reduces parents' labor supply by 27% in period 4.

The wage structure determines the effect of delaying the mandatory retirement age. In an economy with rapid human capital growth, younger generations have higher human capital level and wage rate than the elders. Elder females reduce their labor supply before the legal retirement age, to provide child care to their grandchildren. Delaying retirement age only has limited effects on females' labor supply. In an economy with slow human capital growth, the wide wage gap between old and young recedes. Delaying the retirement age removes the retired grandparents from child care, and parents decrease their labor supply to provide child care. Delaying the retirement age has great effects on reducing young females' labor supply.

8.3 Inheritance Tax

This section states the effect of inheritance tax on household labor supply and saving decisions. In the benchmark setting, households face zero inheritance tax[40]. Keep the other parameters and setting unchanged, I estimate the households' responses with the 30% inheritance tax.

Inheritance tax increases grandparents' savings in no-commitment cases. Figure 12 shows the simulated effects of inheritance tax on saving and labor supply. As the death rate before the period 4 is small, bequest is not likely to occur during these periods. The inheritance law only has few effects on households' saving labor supply behavior. However, after that period, with fewer

adding "several months every year" to the age which pension payments can begin. (Shi, Xu, Zhang & Yao 2015)

[39] The structure experiment uses the wage structure of 2000-2004, in which 50 years old has the highest income rate among all ages.

[40] China proposed inheritance tax law in 2004 but has not yet been able to introduce it due to widespread opposition.

bequest from grandparents, parents provide less elder care and transfer than the benchmark case. The influence of intergenerational relationships on labor supply is less than that in the benchmark case. The policy increases parents' labor supply by 9%. Taxation weakens the crowd out effects of bequest on saving, therefore parents save about 17% more than the benchmark case. Inheritance tax also reduces grandparents' expected consumption level by reducing the exchanged support and care service from parents. Grandparents save about 14% more than the benchmark.

By weakening the bequest incentives, inheritance taxes affects households' saving behavior and indirectly affects labor supply decisions. Bequests give incentives for parents to transfer money and provide elder care to grandparents. Inheritance tax reduces the net value of the bequest in exchange for parents' support and transfer. It causes less transfer and elder care from parents to grandparents. The tax indirectly increases the labor supply of parents at the later stage of the relationship. Grandparents increase their saving rate to maintain bequest incentives. It may offset part of the direct labor supply increasing effects caused by inheritance tax. In contrast to the no-commitment case, inheritance tax increases the intensity of transfer and support in the full-commitment case. In the full-commitment case, grandparents prefer to transfer all the money to parents before death to avoid inheritance tax. Without incentive problems, inheritance tax increases the transfer from grandparents to parents at the early stages and the transfer from parents to grandparents at the late stages. Grandparents save more to sustain the bequest incentive for parents to provide transfer and support after introducing the inheritance tax.

9 Conclusion

This article develops a non-altruistic dynamic contract to analyze how intergenerational relationships affect households' labor supply and saving behavior in the presence of idiosyncratic income shocks and death uncertainty. A distinguishing feature of the model is the use of economic benefits only to sustain intergenerational relationships. From closed form equilibrium allocations, it is straightforward to derive rule of income and labor service allocations within the relationship across time. This article adopts a first step toward showing how do the economic factors of intergenerational relationships affect households' behavior throughout a life-cycle. The framework can be extended to incorporate fertility decisions and life cycle human capital development. The theoretical framework sheds light on a range of questions in which marriage, child care, elder care and fertility are central to the analysis.

My empirical results suggest that intergenerational relationships increase young females' labor supply by 32% and decrease elder females' labor supply by 21% in China. The choices of support and transfer depend on wage structure and market care service price on extensive margins. Households' savings and wage rates determine the intensity of support and transfer of intensive margins. The rapid human capital level growth contributes to the strong intergenera-

tional relations and high labor market participation rate in China. The article ignores human capital development and migration decisions for simplification purposes; this condition may underestimate the effects of intergenerational relationships on labor supply. Strong intergenerational relationships contribute the over-investment on children's education and large scale temporary migration in China. Studying the effects of intergenerational relationships on human capital investment and migration decisions is promising direction for future research.

A further step I take is to quantify the spillover effects of public policies through intergenerational relationships. I discover that child care subsidies increase the labor supply of grandparents and that elder care subsidies also increase the labor supply of parents. Inheritance taxes increase households' savings rate by reducing the bequest incentives. Delaying mandatory retirement age only has limited effects on female labor supply when grandmothers are the primary child care providers, but has big effects when mothers are the primary child care providers. These findings illustrate the importance of modeling intergenerational relationships and household decisions simultaneously. Ignoring the connections between households of different generations can lead to incomplete forecasts of the effects of some policy changes.

References

[1] Barro, R.J. (1974). Are government bonds net wealth?. Journal of Political Economy 82, 1095-1117.

[2] Bauernschuster, S., and Schlotter, M. (2015). Public child care and mothers' labor supply-Evidence from two quasi-experiments. Journal of Public Economics, 123, 1-16.

[3] Battistin, E., De Nadai, M., & Padula, M. (2015). Roadblocks on the Road to Grandma's House: Fertility Consequences of Delayed Retirement.

[4] Becker, G.S. (1974). A theory of social interactions. Journal of Political Economy 82, 1063-1093.

[5] Blau, F. D., and Kahn, L. M. (2005). Changes in the labor supply behavior of married women: 1980-2000 (No. w11230). National Bureau of Economic Research.

[6] Bonsang, E. (2007). How do Middle-Aged Children Allocate Time and Money Transfers to their Older Parents in Europe? Empirica, 34, 171–188.

[7] Chen, F., and Liu, G. (2009). Population aging in China. In International handbook of population aging (pp. 157-172). Springer Netherlands.

[8] Chen, F., Liu, G., and Mair, C. A. (2011). Intergenerational ties in context: Grandparents caring for grandchildren in China. Social Forces, sor012.

[9] Chen, Y., Jin, G. Z., and Yue, Y. (2010). Peer migration in China (No. w15671). National Bureau of Economic Research.

[10] Cigno, A. (2006). A constitutional theory of the family. Journal of Population Economics, 19(2), 259-283.

[11] Cong, Z., and Silverstein, M. (2012). Caring for grandchildren and intergenerational support in rural China: a gendered extended family perspective. Ageing and Society, 32(03), 425-450.

[12] Compton, J., and Pollak, R. A. (2014). Family proximity, child care, and women's labor force attachment. Journal of Urban Economics, 79, 72-90.

[13] Feng, Z., Liu, C., Guan, X., and Mor, V. (2012). China's rapidly aging population creates policy challenges in shaping a viable long-term care system. Health Affairs, 31(12), 2764-2773.

[14] Geurts, T., van Tilburg, T., Poortman, A. R., and Dysktra, P. A. (2015). Child care by grandparents: Changes between 1992 and 2006. Ageing and Society, 35(06), 1318-1334.

[15] Hamaaki, J., Hori, M., and Murata, K. (2014). Intergenerational Transfers and Asset Inequality in Japan: Empirical Evidence from New Survey Data. Asian Economic Journal, 28(1), 41-62.

[16] Hank, K., and Buber, I. (2009). Grandparents caring for their grandchildren findings from the 2004 survey of health, ageing, and retirement in Europe.Journal of Family Issues, 30(1), 53-73.

[17] Ho, C. (2015). Grandchild care, intergenerational transfers, and grandparents' labor supply. Review of Economics of the Household, 13(2), 359-384.

[18] Hsieh, H. L., Chou, S. Y., Liu, E., and Lien, H. M. (2015). Strengthening or Weakening? The Impact of Universal Health Insurance on Intergenerational Coresidence in Taiwan. Demography, 52(3), 883-904.

[19] Jellal, M., and Wolff, F. C. (2005). Free entry under uncertainty. Journal of Economics, 85(1), 39-63.

[20] Johnson, R. W., and Sasso, A. T. L. (2006). The impact of elder care on women's labor supply. The Journal of Health Care Organization, Provision, and Financing, 43(3), 195-210.

[21] Judd, K. L. (1998). Numerical methods in economics. MIT press.

[22] Keane, M. P., and Wolpin, K. I. (1997). The career decisions of young men.Journal of political Economy, 105(3), 473-522.

[23] Kocherlakota, N. R. (1996). Implications of efficient risk sharing without commitment. The Review of Economic Studies, 63(4), 595-609.

[24] Lockwood, L. M. (2014). Incidental bequests: Bequest motives and the choice to self-insure late-life risks (No. w20745). National Bureau of Economic Research.

[25] Luo, Y., LaPierre, T. A., Hughes, M. E., and Waite, L. J. (2012). Grandparents providing care to grandchildren a population-based study of continuity and change. Journal of Family Issues, 33(9), 1143-1167.

[26] McDonald, K. B., and Armstrong, E. M. (2001). De-Romanticizing Black Intergenerational Support: The Questionable Expectations of Welfare Reform. Journal of Marriage and Family, 63(1), 213-223.

[27] McGarry, K. (2000). Testing parental altruism: Implications of a dynamic model (No. w7593). National Bureau of Economic Research.

[28] Melenberg, B., and Zheng, J. (2012). Health Expectancy of the Chinese Elderly: Current Trends and Future Projection. Tilburg University.

[29] National Survey Research Center (2014). China Longitudinal Aging Social Survey. Beijing, China.

[30] National Bureau of Statistics. (2016). National data. [Data file]. Retrieved from: http://data.stats.gov.cn/easyquery.htm?cn=C01

[31] Phelps, E. S., & Pollak, R. A. (1968). On second-best national saving and game-equilibrium growth. The Review of Economic Studies, 35(2), 185-199.

[32] Posadas, J., and Vidal-Fernandez, M. (2012). Grandparent's child care and female labor force participation. IZA discussion paper no. 6398.

[33] Schanzenbach, M. M., and Sitkoff, R. H. (2008). Bequest Puzzles in Economics: A Legal Perspective. In American Law and Economics Association Annual Meetings (p. 96).

[34] Silverstein, M., Cong, Z., and Li, S. (2006). Intergenerational transfers and living arrangements of older people in rural China: Consequences for psychological well-being. The Journals of Gerontology Series B: Psychological Sciences and Social Sciences, 61(5), S256-S266.

[35] Shi R, Xu H, Zhang B, and Yao J. (2015). Slowly, China Prepares to Raise Retirement Age. Retrieved from: http://english.caixin.com/100796734.html

[36] Spear, S. E., & Srivastava, S. (1987). On repeated moral hazard with discounting. The Review of Economic Studies, 54(4), 599-617.

[37] Sugawara, S., and Nakamura, J. (2014). Can formal elderly care stimulate female labor supply? The Japanese experience. Journal of the Japanese and International Economies, 34, 98-115.

[38] Stark, O.,and Zhang, J. (2002). Counter-compensatory inter vivos transfers and parental altruism: Compatibility or orthogonality. Journal of Economic Behavior and Organization 47, 19–25.

[39] The World Bank. (2016). Real Interets Rate [Data file]. Retrieved from: http://data.worldbank.org/indicator/FR.INR.RINR

[40] The World Bank. (2015). Fertility rate. [Data file]. Retrieved from: http://data.worldbank.org/indicator/SP.DYN.TFRT.IN

[41] Trinh, T. (2006). China's pension system: Caught between mounting legacies and unfavourable demographics. Deutsche Bank research, February, 17

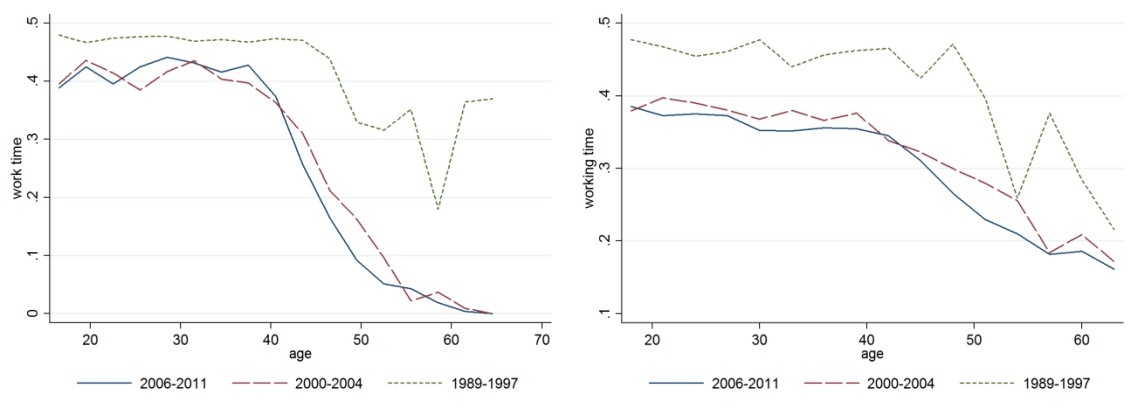
[42] Wang, D. (2006). China's urban and rural old age security system: Challenges and options. China and World Economy, 14(1), 102-116.

[43] Yu, Y. and Liu, S. (2010). Holding up half the sky? Are Chinese women given equal rights in political participation?, in IDAS Symposium: The rising Asia Pacific Region: Opportunities and challenges for cooperation, pp. 300-315.

[44] Zhang, Y., and Maclean, M. (2012). Rolling back of the state in child care? Evidence from urban China. International Journal of Sociology and Social Policy, 32(11/12), 664-681.

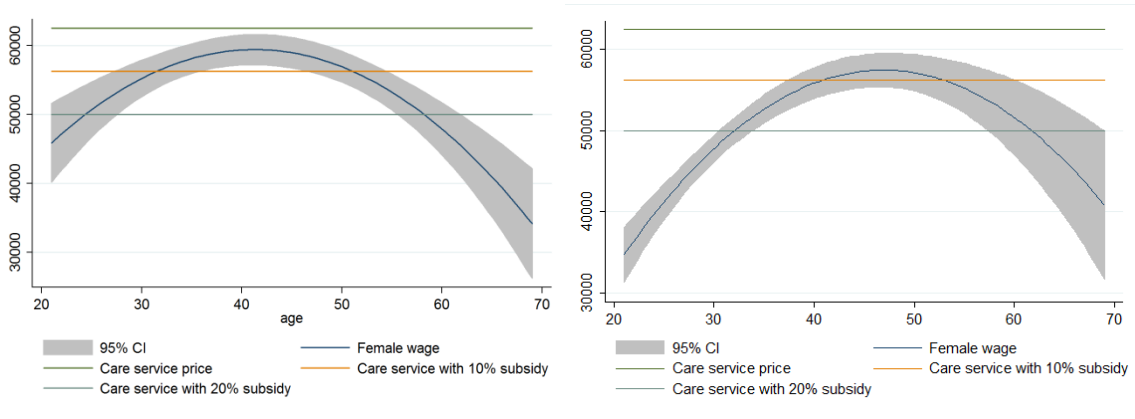
Figures and Tables

Figure 1Female working time by age



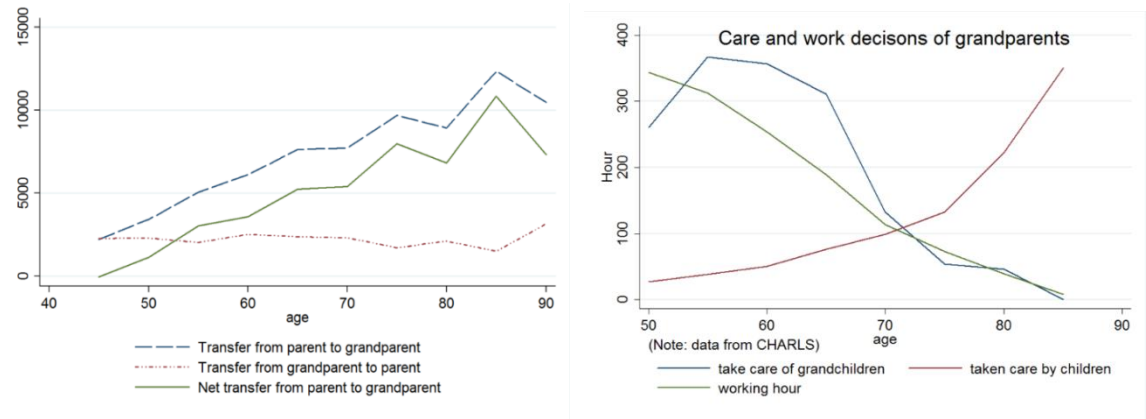
Note: Data from CHNS. The first graph is the female working time of urban area. The second one is the female working time of rural area. In the figures above, I normalize the overall time equal to 1. Assume each individual can have 14 hours to work at most each day. I define working time = (annual work month/12) × (daily work hour/14) × (weekly work day/7).

Figure 2 Wage structure by age cohort in simulation



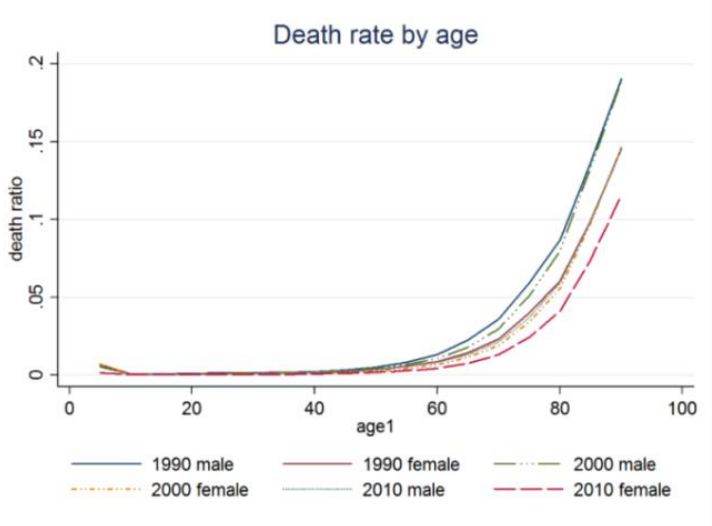
Note: The money unit is Yuan. The first graph represents the benchmark wage structure. I get the wage structure from CHNS data 2006-2011. The average wage rates of two households are always smaller than the care service. The second graph is the wage structure 2. In the wage setting, the wage growth rate by age is from the wage structure of 2000-2004.

Figure 3 Average income transfer and support by age



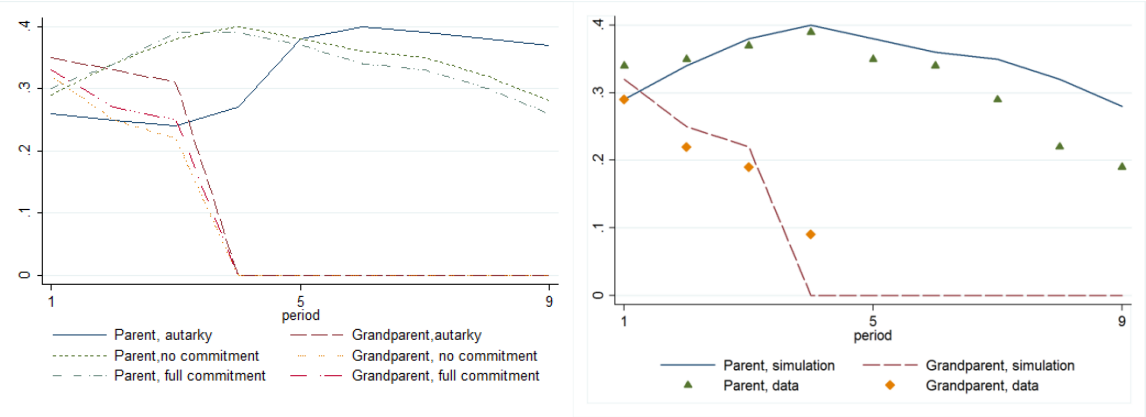
Note: Data from CHARLS 2008-2013. The first graph is the money transfer decisions. The second graph is the support decisions. The money unit is Yuan. Age is grandparents' age. Transfer is the sum value of the gift, regular monetary in-kind support, and non-regular monetary in-kind support. The care hour is the average hour to take care of grandchildren or taken care by children per year. The working hour is the average working hour of grandparents.

Figure 4 Average death rate by age



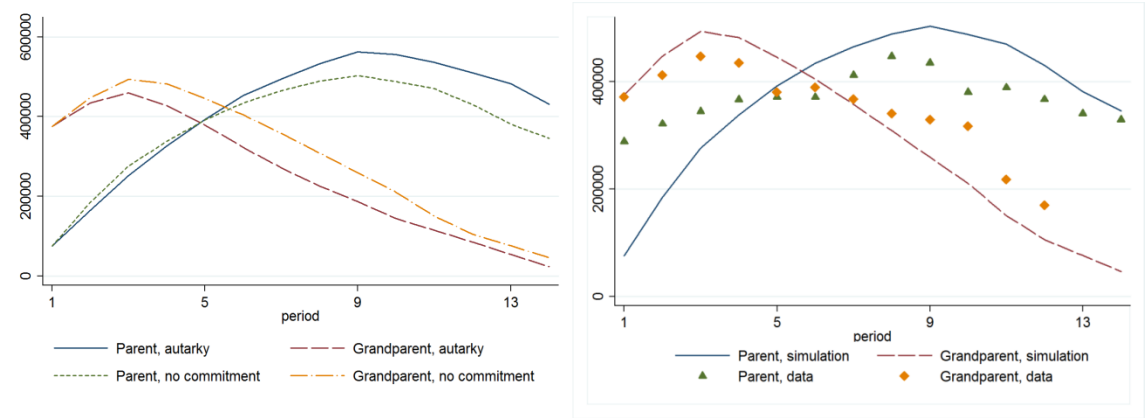
Note: The death rates are from the National Population and Reproductive Health Science Data Center of China. The value is the average death rate of the age cohort.

Figure 5 Simulation results: working hour by age



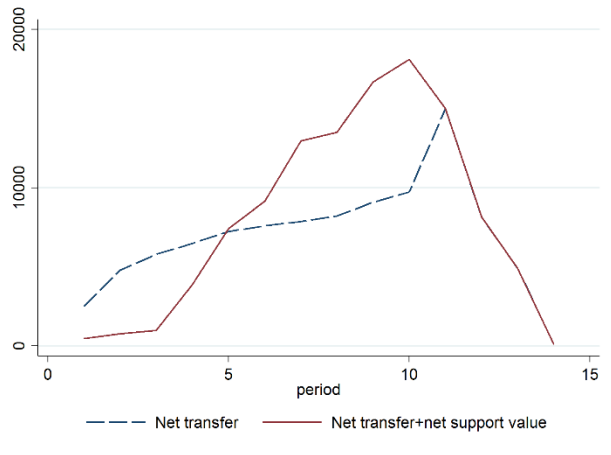
Note: Data from CHNS. Results of the first graph are from simulation. The parameters are following benchmark setting: care service subsidies are 0; inheritance tax is 0; mandatary retirement age is 10; the wage rate is given by wage structure 1 (shown in the first graphs of Figure 1); 42% are type 1 and 58% of households are type 2; grandparents' initial saving is 375,000; and I normalize the overall time to 1. The second graph compares the labor supply by age from simulation and data. The simulation result is using a contract with benchmark setting.

Figure 6 Simulation results: Average asset level by age



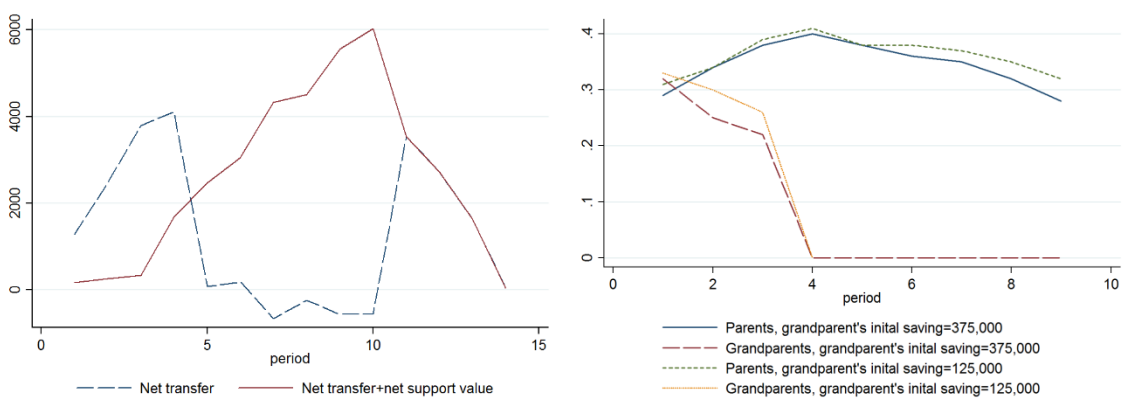
Note: Money unit is Yuan. The parameters are following benchmark setting: care service subsidies are 0; inheritance tax is 0; mandatary retirement age is 10; the wage rate is given by wage structure 1 (shown in the first graphs of Figure 1); 42% are type 1 and 58% of households are type 2; grandparents' initial saving is 375,000; and I normalize the overall time to 1. The first graph is the simulation results of the asset level by age. The second graph compares the asset level by age from simulation and data. The data is from CHNS 2006-2011. The simulation result is using a contract with benchmark setting.

Figure 7 Simulation results: Net transfer from parent to grandparent



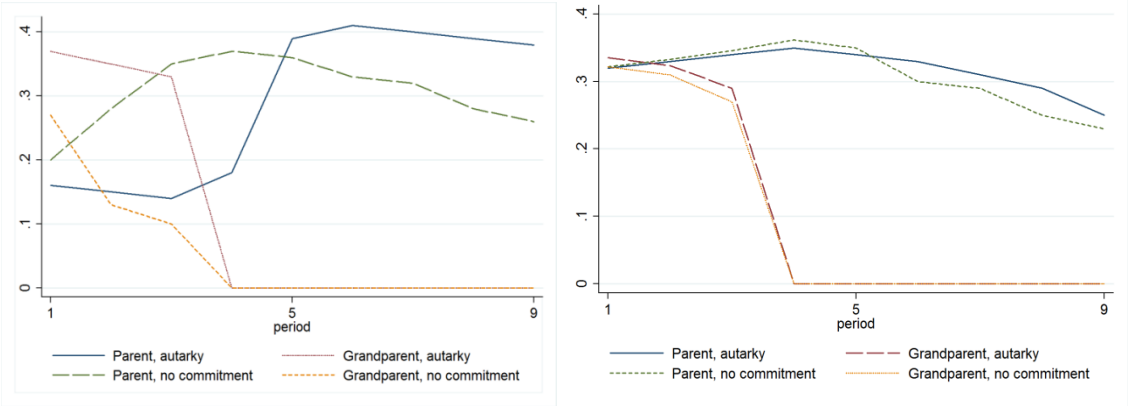
Note: The money unit is Yuan. Transfer and care support value is defined by net transfer plus labor support value. Labor support value is defined by the support hour time support opportunity cost. Money unit is Yuan. The parameters are following benchmark setting: care service subsidies are 0; inheritance tax is 0; mandatary retirement age is 10; the wage rates are given by wage structure 1 (shown in the first graphs of Figure 1); 42% are type 1 and 58% of households are type 2; grandparents' initial saving is 375,000; and I normalize the overall time to 1.

Figure 8 Experiment: grandparent initial saving is 125,000



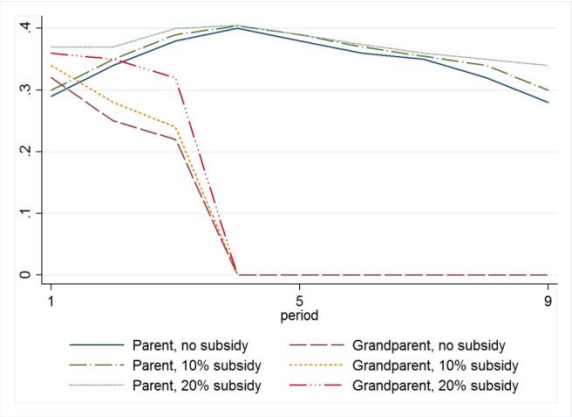
Note: The money unit is yuan. In this part, I have changed the average initial savings of grandparents from 375,000 in benchmark to 125,000. The first graph is transfer value information. The second graph is the labor supply information. Except grandparents' initial savings, the rest of parameters is following benchmark settings.

Figure 9 Simulation results: Labor supply of different types of households



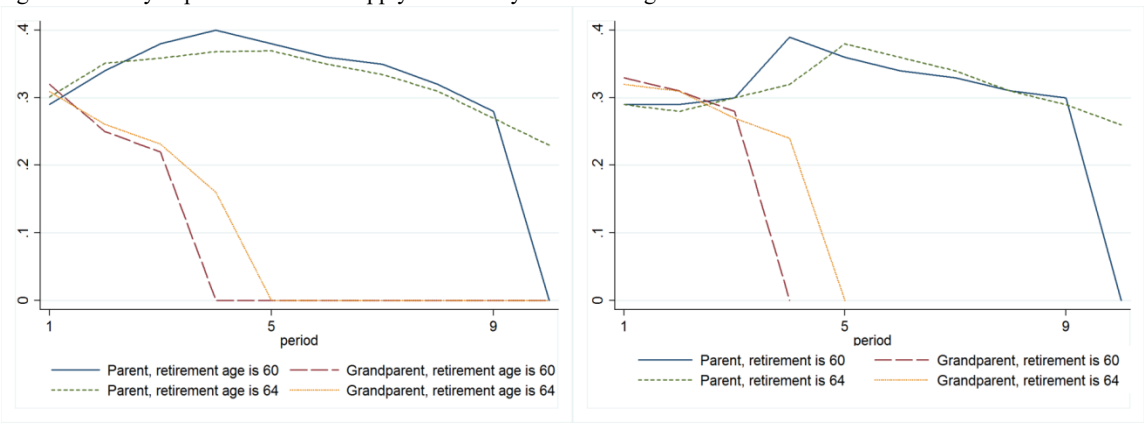
Note: I compare the labor supply of the households with different types of preference on child care and elder care. In Graph 1, both households are type1. In Graph 2, both households are type2. The money unit is Yuan. In the contract, 33.64% of the pair of parents and grandparents is both type 1 preference; and 17.64% are both type 2 preferences. Except the preference setting, the rest of parameters is following benchmark settings.

Figure 10 Policy experiment: Child/elder care subsidy and labor supply



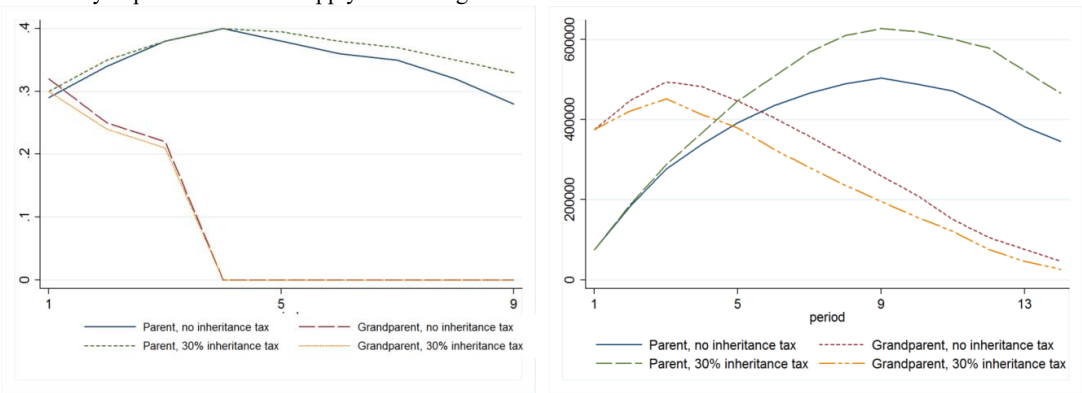
Note: Results from simulation. Both households are in a contract. There are care service subsidies on both child care and elder care service. Except for the car service subsidies setting, the rest of parameters is following benchmark settings.

Figure 11 Policy experiment: Labor supply after Delay retirement age



Note: Results from simulation. Both households are in a contract. In this part, I move the mandatory retirement age from 9 in benchmark to 10. Both households retire later for one period. In the first graph, except for the setting of mandatory retirement age, the rest of parameters is following benchmark settings. In the second graph, the wage rates are given by wage structure 2. Except for the setting of mandatory retirement age and wage rates, the rest of parameters is following benchmark settings.

Figure 12 Policy experiment: Labor supply and saving with Inheritance tax



Note: Results from simulation. Both households are in a contract. In this part, I move the mandatory retirement age from 9 in benchmark to 10. Both households retire later for one period. Except for the setting of mandatory retirement age, the rest of parameters is following benchmark settings. The first graph shows the result of the labor supply. The second graph shows the result of the savings.

Table 1 Individuals and Household characteristics

Variable	Mean	SD	N	Variable	Mean	SD	N
CHNS Data							
working (61>age>24)	62%	0.49	11906	working time	0.34	0.13	8927
education year (61>age>24)	7.54	4.67	11906	annual HH income	35378	44957	10102
HH food and durable good consumption (age>45)	20161	30270	20161	working income rate	33110	41474	8927
HH food and durable good consumption	31774	60717	53483	working income	11902	12467	8927
child care time	0.45	0.51	17128	HH net non-working income	27495	36770	31186
house value (age>45)	402506	724529	7967	Leisure	0.43	0.37	79613
house value	364609	684929	21366	Asset level (age>45)	498564	1029226	7967
child taken care by outside service	19%	0.39	8543	Asset level	455223	986722	21366
hour/day taken care by outside service	2.66	4.54	1623	market elder care price rate	24889	14647	1623
CHARLS Data							
education year	4.97	4.33	22768	rural HH	78%		22768
working	0.19	0.39	22768	eldest child's annual income	34908	39943	7142
HH food and durable good consumption	23131	66379	38177	total HH annual income	24618	69178	31186
HH consumption	30316	72578	31186	total HH annual working income	12197	48741	31186
asset level	423270	4470281	9368	total HH health spending	2275	8882	31186
house value	309035	3738449	9368	transfer from child last year	71%		22228
money value from child last year	1537	3477	22228	transfer from parent last year	2%		22228
child care hour last week	29	28	22228	transfer to child last year	31%		22228
money value from other relatives last year	459	1,962	22228	transfer to child last year	26%		22228
parent care hour last week	8.73	26.89	22228	transfer to parent last year, age<60	0.15	0.15	35877
child care hour last week	31	30	22228	market elder care price rate	31903	18647	8877

Note: The money unit is Yuan. I define working as the people have any kind of income from farming, fishing, gardening, and business. I define migration as whether the people live in the same city or town. Retirement income is the sum value of pension and retirement subsidy. Consumption in CHNS data is all the spending on food and durable goods. Consumption in CHARLS data is all the spending on foods, durable good, clothes, traffic and other consumptions. The value unit of asset and house value is yuan. Of CHNS data, an asset is the net sum value of housing asset, fixed assets of production, housing debt and no housing debt. In CHARLS data, an asset is the net sum value of housing asset, fixed assets of production, financial asset, housing debt and no housing debt. Child care data from CHNS 2006-2011. The money unit is Yuan. Transfer and support data are from CHARLS 2008-2013. The money unit is Yuan. Transfer are defined by the gift, regular monetary or in-kind support and non-regular monetary or in-kind support.

Table 2 estimated parameters

Parameter	Definition	Method	Mean	SE
η	Mean utility weight on leisure	Two stage GLS	0.88	(0.02)
ξ^{η}	The variance of utility weight on leisure	Two stage GLS	0.78	
α	Mean utility weight on child care	Two stage GLS	0.31	(0.01)
ξ^{α}	The variance of weight on child care	Two stage GLS	0.12	
γ	Mean utility weight on elder care (64-72)	Two stage GLS	0.06	(0.01)
ξ^{γ}	The variance of utility weight on elder care (64-72)	Two stage GLS	0.04	
γ	Mean utility weight on elder care (73-80)	Two stage GLS	0.20	(0.02)
ξ^{γ}	The variance of utility weight on elder care (73-80)	Two stage GLS	0.05	
γ	Mean utility weight on elder care (after 81)	Two stage GLS	0.52	(0.06)
ξ^{γ}	The variance of weight on elder care (after 81)	Two stage GLS	0.73	
ρ	Type 1 probability	GMM	0.42	
η_1	Type1 utility weight on leisure	GMM	0.75	
η_2	Type2 utility weight on leisure	GMM	0.97	
α_1	Type1 utility weight on child care	GMM	0.73	
α_2	Type2 utility weight on child care	GMM	0.01	
γ_1	Type1 utility weight on child care (64-72)	GMM	0.14	
γ_2	Type2 utility weight on child care (64-72)	GMM	0.00	
γ_1	Type1 utility weight on child care (73-80)	GMM	0.35	
γ_2	Type2 utility weight on child care (73-80)	GMM	0.09	
γ_1	Type1 utility weight on child care (after 81)	GMM	0.81	
γ_2	Type2 utility weight on child care (after 81)	GMM	0.31	
ι_1	Substitution rate of child care from market and relatives	MLE	2.51	(0.79)
ι_2	Substitution rate of elder care from market and relatives	MLE	1.87	(0.54)
Φ	Correlation between mother and grandmother's wage rate	Correlation	0.46	
φ	Correlation between mother and grandmother's initial asset level	Correlation	0.53	
β	Discount factor	Literature	0.97	

R	Real interests rate	From data	1.01	
$\mathbf{w_{p0}}$	Initial wage rate of mother	OLS	55457	(21072)
$\mathbf{w_{g0}}$	Initial wage rate of grandmother	OLS	59519	(22383)
$\boldsymbol{\pi_1}$	Wage growth rate by age	OLS	0.056	(0.04)
$\boldsymbol{\pi_2}$	Wage growth rate by age square	OLS	-0.0008	(0.00006)

Note: The data estimating income, child care and leisure parameters are from CHNS 2006-2011. Data estimating elder care parameters are from CHARLS 2008-2011. The money unit is Yuan. I use GMM methods to estimate the two type preferences. The interest rate is taken from World Bank, which is the average interest rate from 2006 to 2012.

Table 3 Actual vs. predicted choices and select measures

	Data	Contract	Autarky case
Parents' working time	0.36	0.34	0.33
Parent's working time (before 40)	0.37	0.36	0.27
Parent's working time (After 40)	0.35	0.34	0.38
Parents' leisure	0.43	0.44	0.37
Grandparents' working time	0.23	0.27	0.34
Grandparents' leisure	0.47	0.49	0.45
Child care from outside service	19%	17%	26%
Child care by grandparents	56%	62%	0
Elder care from parents	58%	63%	0
Elder care from outside service	7%	9%	100%
Transfer from parents last year	31%	34%	0
Transfer to parents last year	20%	13%	0
Grandparents' saving	423,270	378,515	318,080
Parents' saving	375,412	345,710	367,777

Note: The information of working, leisure, child care, and saving of data is from CHNS. Elder care information is taken from CHARLS. The prediction values are taken from simulation. Money unit is Yuan. The parameters are following benchmark setting: care service subsidies are 0; inheritance tax is 0; mandatary retirement age is 10; the wage rate is given by wage structure 1 (shown in the first graphs of Figure 1); 42% are type 1 and 58% of households are type 2; grandparents' initial saving is 375,000; and I normalize the overall time to 1. The second graph compares the labor supply by age from simulation and data. The simulation result is using a contract with benchmark setting